# Lower bounds for existential $k$-pebble games

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The existential $k$-pebble game characterizes the expressive power of the existential-positive $k$-variable fragment of the infinitary logic $L_{\infty\omega}$. The winner of the existential $k$-pebble game on two given finite structures can easily be determined in time $O(n^{2k})$. We show that there is no $O(n^{(k-3)/12})$ time algorithm that decides which player can win the existential $k$-pebble game on two given structures. This lower bound is unconditional and does not rely on any unproven complexity theoretic assumptions.

The $k$-consistency test is a well-known heuristic for solving the constraint satisfaction problem (CSP). By the game characterization of Kolaitis and Vardi our result implies that there is no $O(n^{(k-3)/12})$ time algorithm that decides if a given CSP-instance passes the $k$-consistency test.