On the Ordered Conjecture

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Abstract

It is well-known that least fixed-point logic LFP captures the complexity class PTIME on ordered structures. The ordered conjecture claims that LFP is more expressive than first-order logic FO (in short, LFP $>$ FO) on every infinite class O of finite ordered structures. We present two methods which yield that LFP $>$ FO on various types of classes of ordered structures. The first method, the *model-checking method*, among others, can be applied for all such classes O of bounded cliquewidth. By the second method, *the padding method*, we show that for classes O of "bounded treewidth," more precisely, for classes O such that there is a bound for the treewidth of the successor structures associated with the members of O, even DTC $>$ FO on O, where DTC denotes the deterministic transitive closure logic, a logic that captures the complexity class L on ordered structures. Furthermore, with the padding method we get that for every infinite class of ordered structures O we have DTC $>$ FO on the class of all ordered sums $\mathcal{A} \oplus \mathcal{B}$ with $\mathcal{A}, \mathcal{B} \in$ O.

Under some complexity theoretic assumption, we prove the existence of a class O of ordered structures such that on O not only LFP $>$ FO but even LFP has the expressive power of existential second-order logic Σ_1^1.