Regular tree languages, cardinality predicates, and addition-invariant FO

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We consider the logic FO_{card}, i.e., first-order logic with *cardinality predicates* that can specify the size of a structure modulo some number. We study the expressive power of FO_{card} on the class of languages of ranked, finite, labelled trees with successor relations.

Our first main result characterises the class of FO_{card}-definable tree languages in terms of algebraic closure properties of the tree languages. As it can be effectively checked whether the language of a given tree automaton satisfies these closure properties, we obtain a decidable characterisation of the class of regular tree languages definable in FO_{card}.

Our second main result considers first-order logic with unary relations, successor relations, and two additional designated symbols $<$ and $+$ that must be interpreted as a linear order and its associated addition. Such a formula is called *addition-invariant* if, for each fixed interpretation of the unary relations and successor relations, its result is independent of the particular interpretation of $<$ and $+$. We show that the FO_{card}-definable tree languages are exactly the regular tree languages definable in addition-invariant first-order logic.