

# (Tree-) automatic well-founded order trees have small ordinal ranks

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Since well-founded order trees (wf-trees) are special well-founded partial orders, each wf-tree has an (ordinal) rank which is the transfinite notion of the depth of the tree. We show that string-automatic wf-trees have rank bounded by  $\omega^2$  and tree-automatic wf-trees have rank bounded by  $\omega^\omega$ . The proof relies on Delhomme's decomposition technique for (tree-)automatic structures. We apply an improved version of Delhomme's technique to the so called erank of well-founded trees. The erank of a wf-tree is the rank of the subtree of its infinitely branching nodes. We obtain the result that the erank of tree-automatic wf-trees is strictly bounded by  $\omega^\omega$ . For string-automatic wf-trees it is known that the erank is bounded by  $\omega$ . We obtain our main result by proving that the rank and erank of any wf-tree differ at most by a factor  $\omega$ . As a corollary, we obtain that the isomorphism problem for tree-automatic wf-trees is in level  $\Delta_{\omega^\omega}^0$  of the hyperarithmetical hierarchy (in fact, it is complete for this level under Turing reductions).