

Where First-Order and Monadic Second-Order Logic Coincide

Martin Grohe
Humboldt-Universität zu Berlin

We study on which classes of graphs first-order logic (FO) and monadic second-order logic (MSO) have the same expressive power. We show that for all classes C of graphs that are closed under taking subgraphs, FO and MSO have the same expressive power on C if, and only if, C has bounded tree depth. Tree depth is a graph invariant that measures the similarity of a graph to a star in a similar way that tree width measures the similarity of a graph to a tree. For classes just closed under taking induced subgraphs, we show an analogous result for guarded second-order logic (GSO), the variant of MSO that not only allows quantification over vertex sets but also over edge sets. A key tool in our proof is a Feferman-Vaught-type theorem that is constructive and still works for unbounded partitions.

(This is joint work with Michael Elberfeld and Till Tantau.)