

Definability of linear equation systems over groups and rings

Joint work with A. Dawar, E. Grädel, B. Holm, E. Kopczynski

One of the major open question in finite model theory is whether there is a logic for PTIME. As one promising candidate, fixed-point logic with counting, FPC, has been studied extensively, and indeed, FPC has been shown to capture PTIME on important classes of structures.

Although Cai, Fürer and Immerman ruled out FPC for the general case already in 1992, it was only in 2007 that Atserias et. al [1] found a class of *natural* problems explaining the major shortcoming of FPC. Specifically, they proved that the important problem of solving linear equation systems (SLES) over finite Abelian groups cannot be expressed in FPC; moreover, all other known queries separating FPC from PTIME turned out to be reducible to SLES via simple logical reductions. These results show that problems from algebra provide a new source of operators which yield logics of polynomial-time data complexity which are strictly more expressive than FPC (cf. the notion of rank logics [2]).

Motivated by these insights, we study SLES over various classes of finite groups and rings from the viewpoint of logical (inter-)definability. All problems that we consider are decidable in polynomial time, but not expressible in FPC. Based on the structure theory of finite rings, we prove that on important classes of rings, SLES can be reduced to SLES over cyclic groups, which constitute the most basic class of domains for SLES. Further, we prove closure properties for classes of queries that reduce to SLES over fixed rings. As an application, these closure properties provide normal forms for logics extended with solvability operators. In my talk I give an overview of the recent advances in this area, focussing on extensions of first-order logic by operators deciding SLES over a fixed ring.

References

- [1] Albert Atserias, Andrei A. Bulatov, and Anuj Dawar. Affine systems of equations and counting infinitary logic. *Theor. Comput. Sci.*, 410(18):1666–1683, 2009.
- [2] Anuj Dawar, Martin Grohe, Bjarki Holm, and Bastian Laubner. Logics with rank operators. In *LICS '09: Proceedings of the 2009 24th Annual IEEE Symposium on Logic In Computer Science*, pages 113–122, Washington, DC, USA, 2009. IEEE Computer Society. ISBN 978-0-7695-3746-7.