The Support of Weighted Unranked Tree Automata

Manfred Droste and Doreen Götze

Institut für Informatik, Universität Leipzig, D-04109 Leipzig, Germany
{droste,goetze}@informatik.uni-leipzig.de

Unranked trees are a common concept in computer science. For instance (fully structured) XML-documents can be seen as unranked trees. With the help of unranked tree automata, one can investigate qualitative questions on XML-documents.

To allow the study of quantitative aspects, Droste and Vogler (TOCS 2011) recently proposed and investigated bottom-up weighted unranked tree automata over semirings. These automata relate each of their states $q \in Q$ and each letter a to a weighted finite string automaton $\mathcal{A}_{q,a}$ over the same semiring and the set Q as alphabet. For the definition of the behavior of weighted unranked tree automata, Droste and Vogler employed two distinct approaches, which for semirings coincide. The first one can be viewed as a conceptual one; it uses runs, which associate each position of an input tree to a state, and proceeds by calling for every position of the tree the weighted string automaton associated with the state and the letter at that position. The second definition is based on extended runs and could be viewed as an operational model; extended runs consist of global trees composed of local runs of the automata associated with the state and the letter assigned to each position of a given input tree.

We wish to investigate the support of weighted unranked tree automata; it is defined as the language of all trees evaluated to non-zero.

We will consider strong bimonoids as weight structures which can be viewed as 'semirings without requiring distributivity'. These form a much larger class of weight structures than semirings. For instance, they contain all bounded (non-distributive) lattices, which occur in multi-valued logics, and also weight structures recently investigated e.g. by Chatterjee, Doyen, and Henzinger for modeling peak power consumption of energy.

The main results of our paper are the following:

- We introduce weighted unranked tree automata over strong bimonoids by using valid extended runs.
- We show that for zero-sum free, commutative strong bimonoids, the support of a weighted unranked tree automaton is recognizable.
- An unranked tree automaton for the support can be effectively constructed if Kirsten's zero generation problem (DLT 2009) is decidable, even if the bimonoid operations are not computable.

For our proofs, we heavily use the methods of Kirsten (DLT 2009), but adjusted for constructing (the technically more involved) unranked tree automata.