Constructions and Algorithms for ω-Automata

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Automata on infinite words, or ω-automata, have their origin as a tool in a decision procedure for the monadic second-order (MSO) logic over the structure $(\mathbb{N}, +1)$ of the natural numbers with the successor function [3]. Each such formula can be translated into a nondeterministic Büchi automaton, which is syntactically the same as a standard nondeterministic finite automaton. However, the semantics refers to infinite words, namely an infinite word is accepted if there is a run that visits an accepting state infinitely often. The satisfiability problem for the MSO formulas then reduces to an emptiness test for the resulting automaton, which can be solved with standard graph algorithms.

With the rising interest in formal methods for verification, Büchi automata came back into focus. In [12] it was shown that linear temporal logic (LTL) can be translated into Büchi automata with only a single exponential blow-up (compared to the non-elementary complexity of the translation from MSO). Since LTL is a popular logic for specifying properties of system executions, Büchi automata have become part of verification tools like SPIN [5].

This new interest also stimulated new research for classical problems like the complementation problem for Büchi automata. As opposed to finite automata, the subset construction does not work for complementation (or determinization), and it can be shown that a blow-up of 2^n is not sufficient for the complementation of n-state Büchi automata [8, 13]. Many constructions and optimizations of existing constructions have been proposed and also evaluated experimentally (see, for example, [6, 9, 4, 2, 11]).

Another interesting logic that can be translated into ω-automata is the first-order (FO) logic over the structure $(\mathbb{R}, \mathbb{Z}, <, +)$, that is, the real numbers with a predicate for integers, the less than relation, and addition. As integers can be seen as finite words (e.g., in their decimal representation) real numbers naturally correspond to infinite words. An interesting aspect of $FO(\mathbb{R}, \mathbb{Z}, <, +)$ is that it can be handled by a subclass of ω-automata, called deterministic weak Büchi automata [1]. Deterministic weak automata have many good properties similar to standard deterministic finite automata on finite words. Most notably, their minimal automata can be characterized using a congruence on finite words [10], and a minimal automaton can be computed efficiently [7].

In this talk I will survey the connections between ω-automata and logics, as well as some of the central constructions and algorithmic problems like complementation and minimization, as discussed above.

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