Size of Unary One-Way Multi-Head Finite Automata

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1 Introduction

One of the main topics of descriptional complexity is the question of how the size of the description of a formal language varies when being described by different formalisms. A fundamental result is the exponential trade-off between the number of states of nondeterministic (NFA) and deterministic finite automata (DFA) (see, for example, [12]). Additional exponential and double-exponential trade-offs are known, for example, between unambiguous and deterministic finite automata, between alternating and deterministic finite automata, between deterministic pushdown automata and DFA, and between the complement of a regular expression and conventional regular expressions. Beside these *recursive* trade-offs, bounded by recursive functions, it is known that there also *non-recursive* trade-offs, which are not bounded by any recursive function. Such trade-offs have at first been shown to exists between context-free grammars generating regular languages and finite automata [12]. For a survey on recursive and non-recursive trade-offs we refer to [3, 5].

Unary languages, that is, languages defined over a singleton alphabet, are of particular interest, since in this case often better or more precise results than in the case of arbitrary alphabets can be obtained. For example, the trade-off of 2^n between an n-state NFA and DFA, is reduced to $e^{\Theta(\sqrt{n \cdot \ln(n)})}$ in the unary case [1]. The descriptional complexity of unary regular languages has been studied in many ways. On the one hand, many automata models such as one-way finite automata, two-way finite automata, pushdown automata, or context-free grammars for unary languages are investigated and compared to each other with respect to simulation results and the size of the simulation (see, for example, [2, 11, 13, 15]). On the other hand, many results concerning the state complexity of operations on unary languages have been obtained (see, for example, [4, 7, 10, 14]).

Here, we consider deterministic one-way multi-head finite automata accepting unary languages. Since it is known that every unary language accepted by a one-way multi-head finite automaton is semilinear and thus regular [6, 16], it is of interest to investigate the descriptional complexity of such devices in comparison with the models mentioned above. In detail, we establish upper and lower bounds for the conversion of k-head DFA to one-head DFA and one-head

NFA. Moreover, we investigate the size costs for simulating one-head NFA by k-head DFA and the computational complexity of decidability questions for k-head DFA. Unary deterministic one-way multi-head finite automata have already been studied in [9]. The main results obtained there are infinite proper hierarchies with respect to the number of states as well as to the number of heads. It should be noted that the trade-offs between general k-head DFA and one-head DFA are non-recursive for all $k \geq 2$ [8].

2 Results

As is often the case in connection with unary languages, the function

$$F(n) = \max\{ \operatorname{lcm}(c_1, c_2 \ldots, c_l) \mid c_1, c_2, \ldots, c_l \geq 1 \text{ and } c_1 + c_2 + \cdots + c_l = n \},$$

which gives the maximal order of the cyclic subgroups of the symmetric group of n symbols, plays a crucial role, where lcm denotes the *least common multiple.*

Theorem 1. *For any integers $k, n \geq 2$ so that n is prime, there is a unary n-state DFA(k) M, such that $n \cdot F(n)^{k-1}$ states are necessary for any DFA to accept the language $L(M)$.*

Theorem 2. *Let $k, n \geq 1$ and M be a unary n-state DFA(k). Then there is a constant t depending only on k so that $O(n \cdot F(t \cdot n)^{k-1})$ states are sufficient for a DFA to accept the language $L(M)$. The DFA can effectively be constructed from M.*

Theorem 3. *Let $k, n \geq 2$ be constants and M be a unary n-state DFA(k). Then $O(n^{2k})$ states are sufficient for an NFA to accept the language $L(M)$. The NFA can effectively be constructed from M.*

Theorem 4. *For any integers $k, n \geq 2$, there is a unary n-state DFA(k) M, such that $\Omega(n^k)$ states are necessary for any NFA to accept the language $L(M)$.*

Theorem 5. *Let $k \geq 1$, $n \geq 2$ be constants, $t = \lfloor \frac{-3+\sqrt{8n+1}}{2} \rfloor$, and M be a unary n-state NFA. Then*

$$n' \leq \begin{cases} n^2 - 2 + F(n), & \text{if } k = 1; \\ n^2 - 2 + \left(n - \frac{t^2+t}{2}\right)^{\lceil \frac{t}{k} \rceil}, & \text{if } 1 < k < t/2; \\ 2n^2, & \text{if } k \geq t/2. \end{cases}$$

states are sufficient for a DFA(k) to accept the language $L(M)$. The DFA(k) can effectively be constructed from M.

Theorem 6. *Let $k \geq 1$ be a constant. For any integer $m \geq 1$ there is an integer $n > m$ and a unary n-state NFA M, such that $c_2 \cdot \sqrt[k]{e^{\frac{\sqrt{2n}}{\sqrt{c_1 \ln(\sqrt{2n})}}}}$ states are necessary for any DFA(k) to accept the language $L(M)$, where $c_1, c_2 > 0$ are two constants.*

Lemma 7. *Let $k \geq 1$ and M be an n-state DFA(k). Then there exists an n-state DFA(k) M' accepting the complement of $L(M)$. The DFA(k) M' can effectively be constructed from M.*

Theorem 8. *Let $k \geq 1$ be an integer. Then the problems to decide emptiness, universality, finiteness, inclusion, and equivalence for unary DFA(k) are LOGSPACE-complete.*

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