Recent advances on valence automata as a generalization of automata with storage

Phoebe Buckheister and Georg Zetzsche

Technische Universität Kaiserslautern
zetzsche@cs.uni-kl.de

Abstract. A valence automaton over a monoid M is a finite automaton in which each edge carries an input word and an element of M. A word is then accepted if there is a run that spells the word such that the product of the monoid elements is the identity.
By choosing suitable monoids M, one can obtain various kinds of automata with storage as special valence automata. Examples include pushdown automata, blind multicounter automata, and partially blind multicounter automata. Therefore, valence automata offer a framework to generalize results on such automata with storage.
This talk will present recent advances in this direction. The addressed questions include: For which monoids do we have a Parikh's Theorem (as for pushdown automata)? For which monoids can we avoid silent transitions?

Let M be a monoid. We define a *valence automaton over* M to be a tuple $A = (Q, X, M, E, q_0, F)$, in which Q is a finite set of *states*, X is an alphabet, $E \subseteq Q \times X^* \times M \times Q$ is is a finite set of *edges*, $q_0 \in Q$ is the *initial state*, and $F \subseteq Q$ is the set of *final states*.

A triple $(q, m, w) \in Q \times M \times X^*$ is called a *configuration*. For configurations $(q, m, w), (q', m', w')$, we write $(q, m, w) \Rightarrow_A (q', m', w')$ if there is an edge $(q, u, n, q') \in E$ such that $w' = wu$ and $m' = mn$. An edge (q, w, m, q') is called a *silent transition* (or λ-*transition*) if $w = \lambda$. The language *accepted by* A is then defined as

$$L(A) = \{w \in X^* \mid \exists q \in F : (q_0, 1, \lambda) \Rightarrow_A^* (q, 1, w)\}.$$

In other words, a word is accepted by A if there is a computation that reads the word and the product of the monoid elements on the edges is the identity. The class of languages accepted by valence automata over M will be denoted by $\mathsf{VA}(M)$. If we forbid silent transitions, the corresponding language class is denoted $\mathsf{VA}^+(M)$.

For many types of automata with storage, there is a monoid M such that valence automata behave like automata of the corresponding type. This is the case, for example, for *pushdown automata*, *blind counter automata*, *partially blind counter automata*, and *Turing machines*. For details on which monoids lead to these automata types, we refer the reader to [8].

Thus, valence automata allow us to ask how the structure of the storage mechanism influences the expressive power and decidability properties of automata: We can study for which monoids classical results about automata with storage still hold. This talk will present recent advances in this direction. The discussed questions include:

- For which monoids M does $\mathsf{VA}(M)$ contain only languages with semilinear Parikh image?
- For which monoids M do we have $\mathsf{VA}(M) \subseteq \mathsf{CF}$?
- For which monoids M can we eliminate λ-transitions, that is, when does $\mathsf{VA}(M) = \mathsf{VA}^+(M)$?
- Which language classes arise as $\mathsf{VA}(M)$?

The results mentioned in the talk appeared in [7, 8, 1].

References

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