Computationally Complete Chains of Evolutionary Processors with Random Context Filters

Bianca Truthe

Otto-von-Guericke-Universität Magdeburg, Germany

truthe@iws.cs.uni-magdeburg.de

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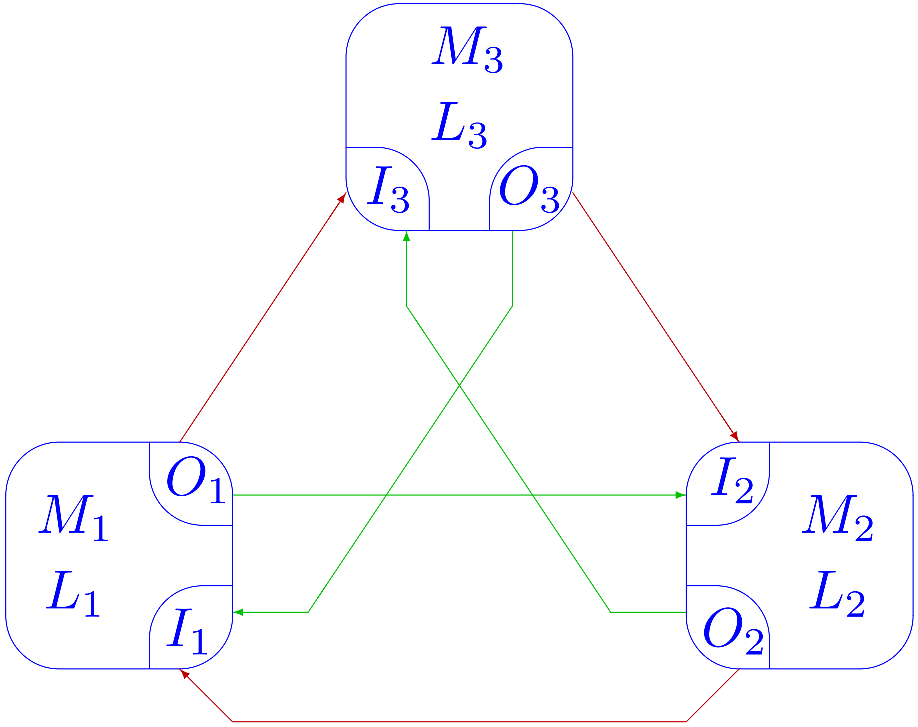
Introduction



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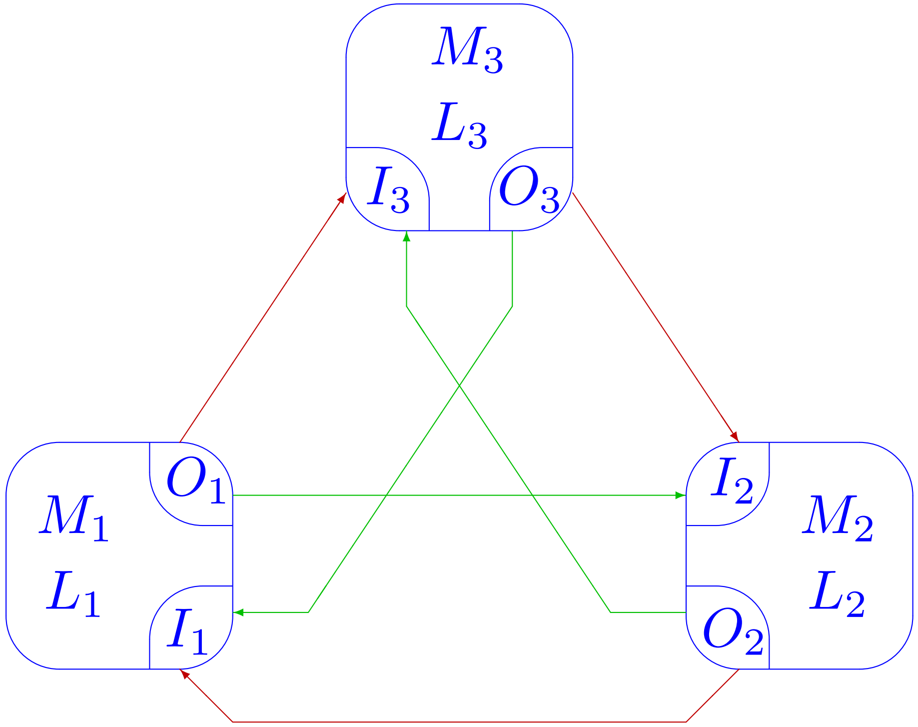

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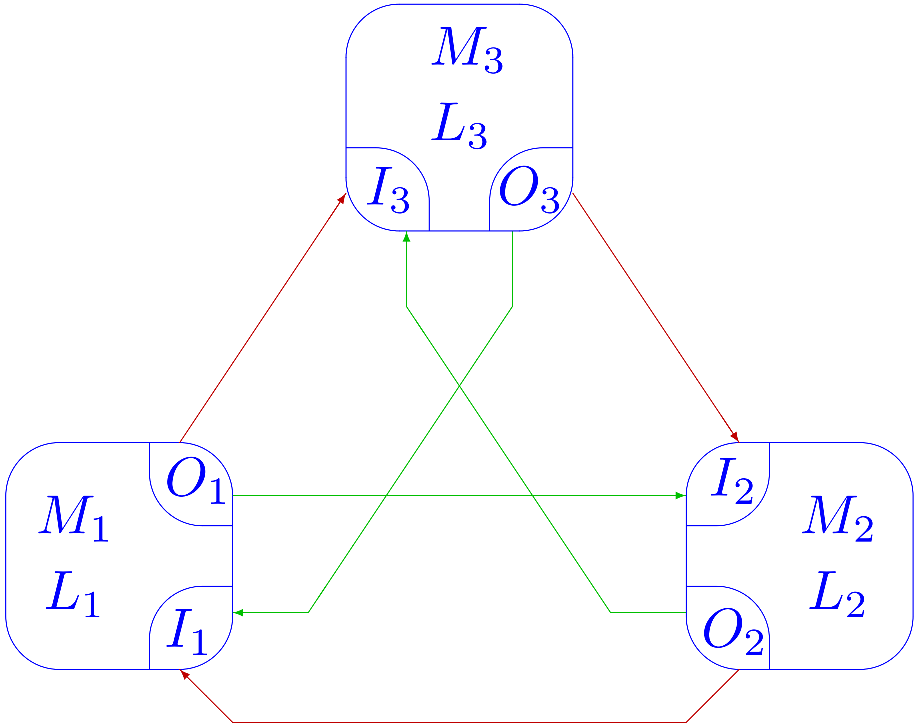

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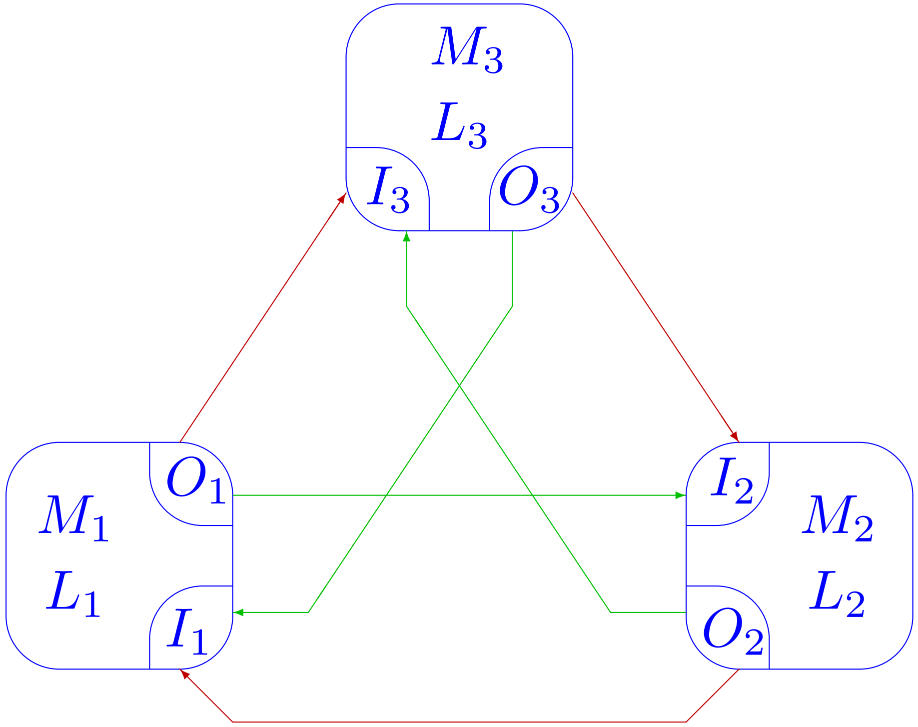

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- J. Dassow, F. Manea: DCFS 2010 $\rightsquigarrow$ stars, wheels, grids

Definitions

ANEP: $\mathcal{N} = (V, U, N_1, N_2, \ldots, N_n, E, j, O)$

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Processor: $N_i = (M_i, PI_i, FI_i, PO_i, FO_i, \beta_i)$

substituting: $M_i \subseteq \{ a \to b \mid a, b \in V \}$

deleting: $M_i \subseteq \left\{ a \xrightarrow{p} \lambda \mid a \in V \right\}$ for $p \in \{ *, r, l \}$

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Language accepted: $L(\mathcal{N}) = \{ w \in V^* \mid \exists t \geq 0\, \exists o \in O : C_t^w(o) \neq \emptyset \}$

Previous Work

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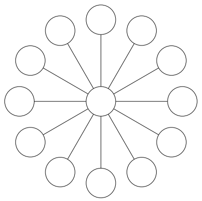
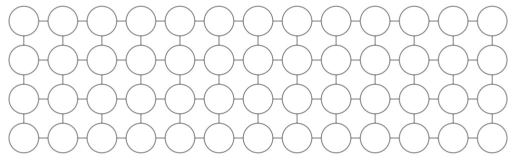


Every recursively enumerable language can be accepted by a star with 13 nodes or a grid with 4×13 nodes.



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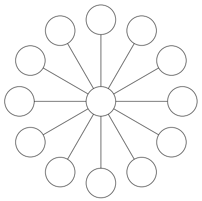

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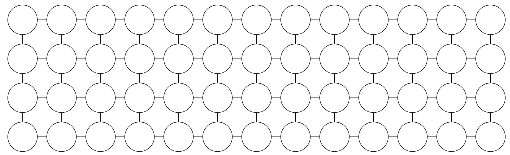
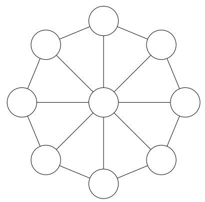


Every 2-tag system can be simulated by a wheel with 9 nodes.



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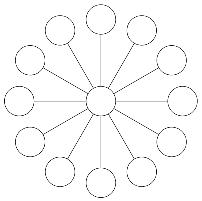

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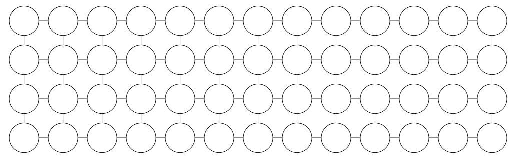

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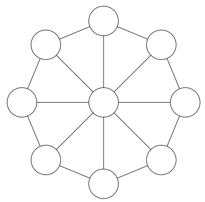

Open: Computational power of wheels and chains.

Chains

Idea: Reverse simulation of a grammar in Kuroda normal form

$A \to \lambda$, $A \to x$, $A \to CD$, $AB \to CD$ with $A, B, C, D \in N$, $x \in N \cup T$

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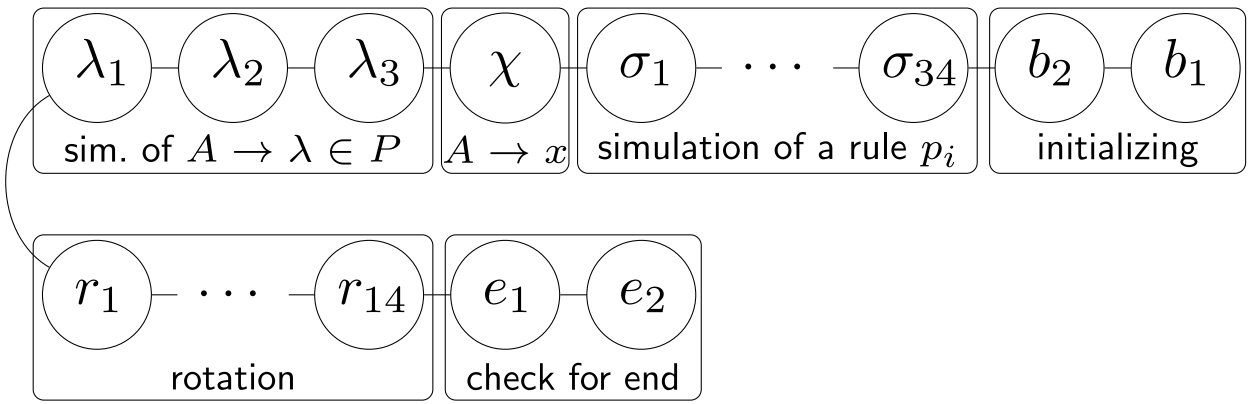
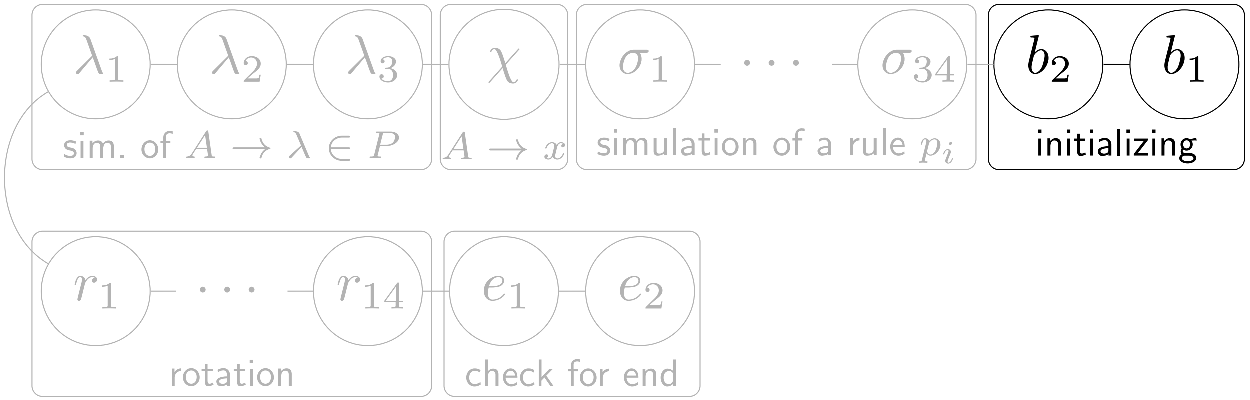


Initialization



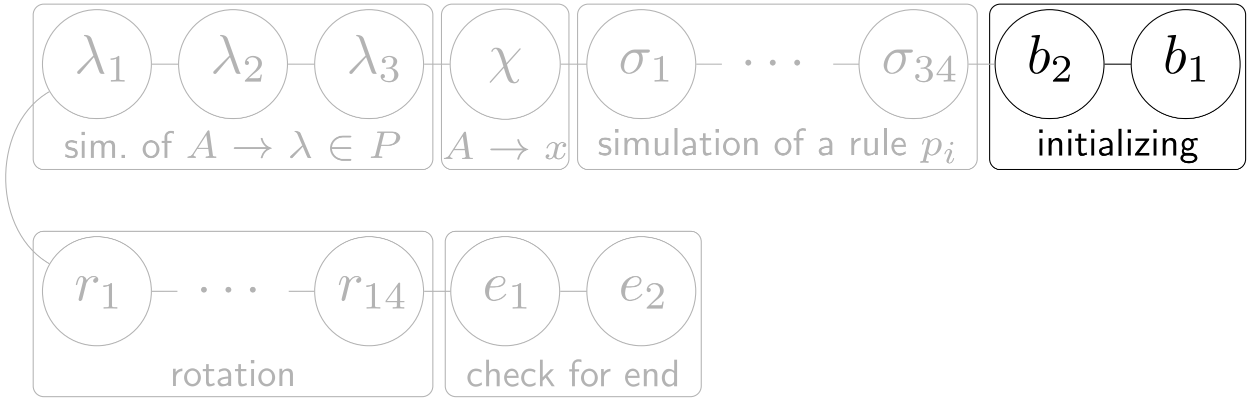
b_1: $\{ \lambda \xrightarrow{r} \xi \}$

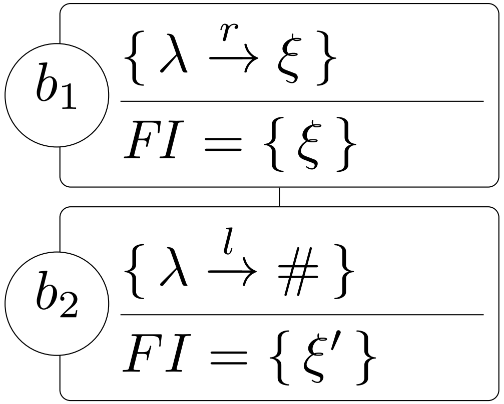
$FI = \{ \xi \}$

b_2: $\{ \lambda \xrightarrow{l} \# \}$

$FI = \{ \xi' \}$

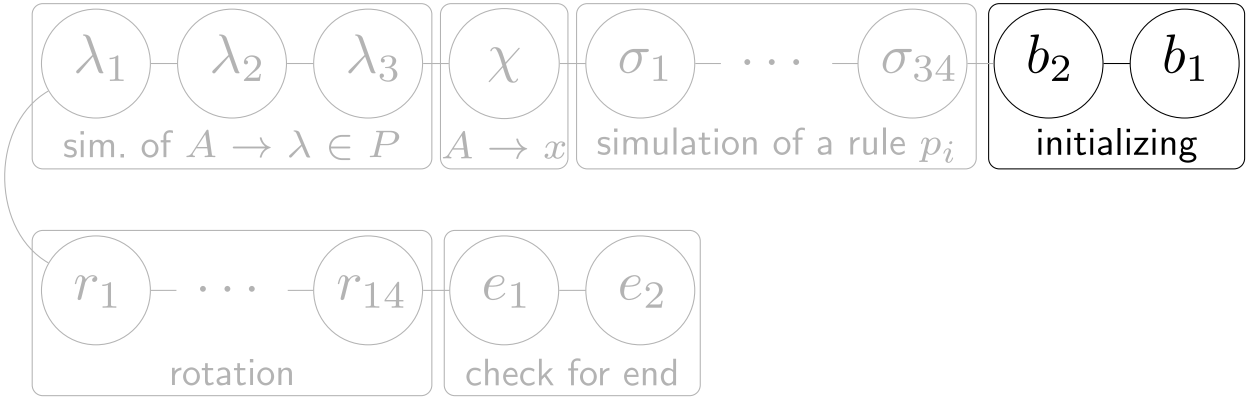
Initialization

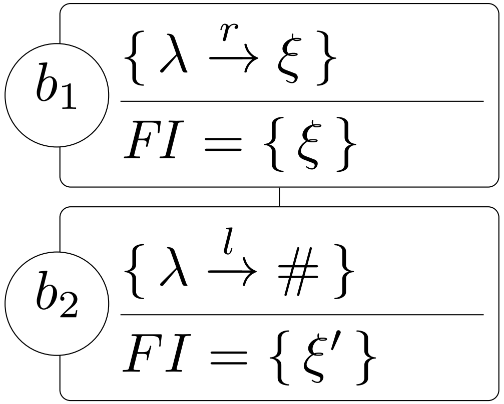




w

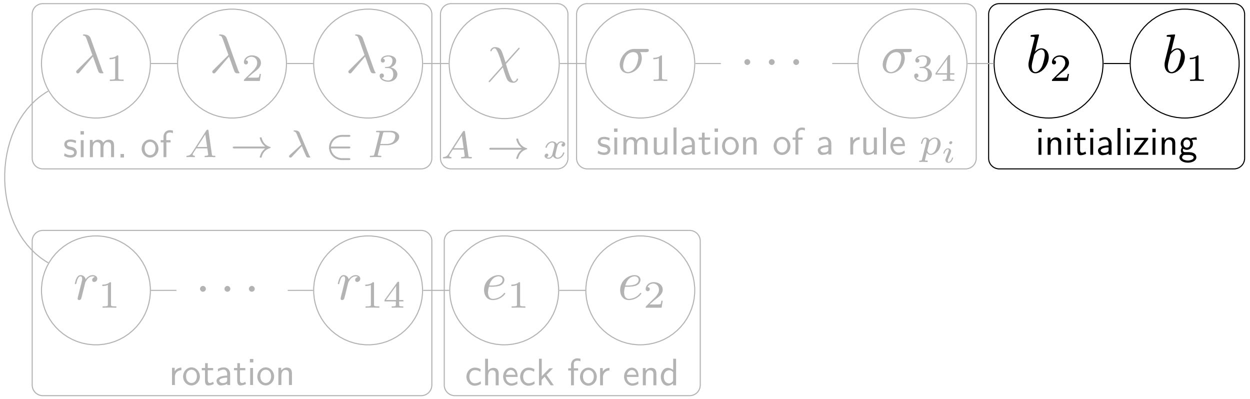
Initialization

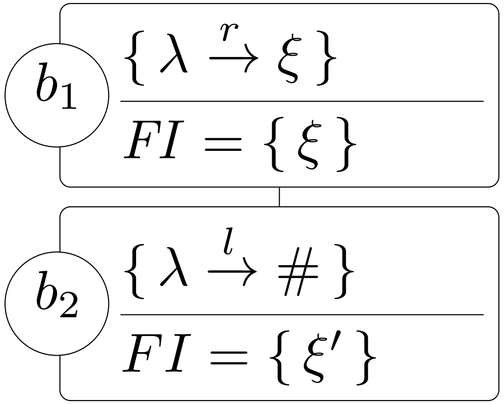




$w\xi$

Initialization





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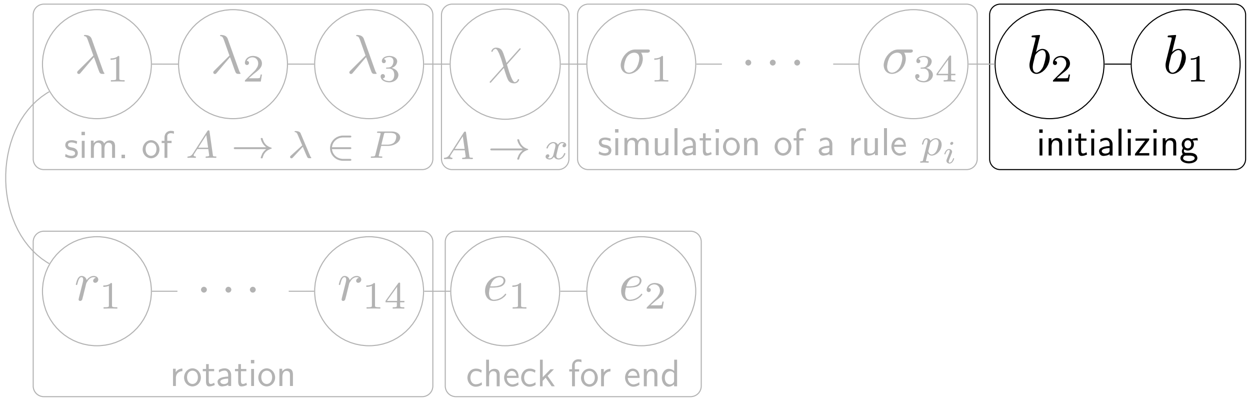

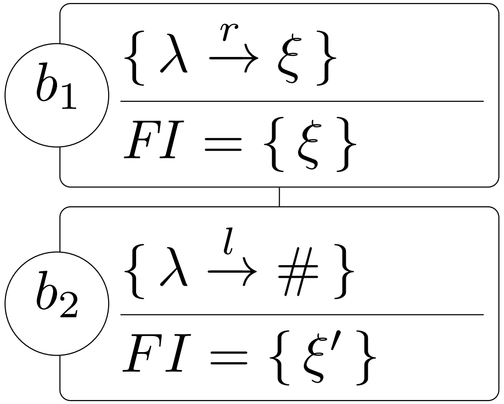

$\#w\xi$

Rotation

r_1: $\{\, [0] \xrightarrow{r} \lambda \,\}$ — $PI = \{\, \xi, \xi' \,\}$

r_2: $\{\, \lambda \xrightarrow{r} [0] \,\}$ — $FI = \{\, [0] \,\}$

r_3: $\{\, \langle i \rangle \to x_i \mid \mu(i) \,\}$ — $FO = \{\, \langle i \rangle \mid \mu(i) \,\}$

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r_5: $\{\, \xi \to \xi' \,\}$ — $FI = \{\, \xi' \,\}$

$x_i w_\xi$

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$$x_i w_\xi [0]$$

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$\langle i\rangle w_\xi [0]$

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$$\langle i \rangle w_{\xi'} [0]$$

Rotation

r_5: $\{\, \xi \to \xi' \,\}$ — $FI = \{\, \xi' \,\}$

r_6: $\{\, [i-1] \to [i]' \mid \mu(i) \,\}$ — $FO = \{\, [i-1] \mid \mu(i) \,\}$

r_7: $\{\, \langle i \rangle \to \langle i-1 \rangle' \mid \mu(i) \,\}$ — $FO = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_8: $\{\, \xi' \to \xi \,\}$ — $FI = \{\, \xi \,\}$

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$$\langle i \rangle w_{\xi'} [1]'$$

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$$\langle i-1 \rangle' w_{\xi'} [1]'$$

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r_6: $\{\, [i-1] \to [i]' \mid \mu(i) \,\}$ — $FO = \{\, [i-1] \mid \mu(i) \,\}$

r_7: $\{\, \langle i \rangle \to \langle i-1 \rangle' \mid \mu(i) \,\}$ — $FO = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_8: $\{\, \xi' \to \xi \,\}$ — $FI = \{\, \xi \,\}$

$$\langle i-1 \rangle' w_{\xi'} [1]'$$

Rotation

r_5: $\{\, \xi \to \xi' \,\}$; $FI = \{\, \xi' \,\}$

r_6: $\{\, [i-1] \to [i]' \mid \mu(i) \,\}$; $FO = \{\, [i-1] \mid \mu(i) \,\}$

r_7: $\{\, \langle i \rangle \to \langle i-1 \rangle' \mid \mu(i) \,\}$; $FO = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_8: $\{\, \xi' \to \xi \,\}$; $FI = \{\, \xi \,\}$

$$\langle i-1 \rangle' w_\xi [1]'$$

Rotation

r_8: $\{ \xi' \to \xi \}$ — $FI = \{ \xi \}$

r_9: $\{ [i]' \to [i] \mid \mu(i) \}$ — $FO = \{ [i]' \mid \mu(i) \}$

r_{10}: $\{ \langle i-1 \rangle' \to \langle i-1 \rangle \mid \mu(i) \}$ — $FI = \{ \langle 0 \rangle \}$, $FO = \{ \langle i-1 \rangle' \mid \mu(i) \}$

r_{11}: $\{ \xi \to \xi' \}$ — $FI = \{ \xi' \}$

$$\langle i-1 \rangle' w_\xi [1]'$$

Rotation

r_8: $\{\, \xi' \to \xi \,\}$

$FI = \{\, \xi \,\}$

r_9: $\{\, [i]' \to [i] \mid \mu(i) \,\}$

$FO = \{\, [i]' \mid \mu(i) \,\}$

r_{10}: $\{\, \langle i-1\rangle' \to \langle i-1\rangle \mid \mu(i) \,\}$

$FI = \{\, \langle 0\rangle \,\}$,
$FO = \{\, \langle i-1\rangle' \mid \mu(i) \,\}$

r_{11}: $\{\, \xi \to \xi' \,\}$

$FI = \{\, \xi' \,\}$

$$\langle i-1\rangle' w_\xi [1]'$$

Rotation

r_8: $\{ \xi' \to \xi \}$
$FI = \{ \xi \}$

r_9: $\{ [i]' \to [i] \mid \mu(i) \}$
$FO = \{ [i]' \mid \mu(i) \}$

r_{10}: $\{ \langle i-1 \rangle' \to \langle i-1 \rangle \mid \mu(i) \}$
$FI = \{ \langle 0 \rangle \}$,
$FO = \{ \langle i-1 \rangle' \mid \mu(i) \}$

r_{11}: $\{ \xi \to \xi' \}$
$FI = \{ \xi' \}$

$\langle i-1 \rangle' w_\xi [1]$

Rotation

r_8: $\{\, \xi' \to \xi \,\}$

$FI = \{\, \xi \,\}$

r_9: $\{\, [i]' \to [i] \mid \mu(i) \,\}$

$FO = \{\, [i]' \mid \mu(i) \,\}$

r_{10}: $\{\, \langle i-1\rangle' \to \langle i-1\rangle \mid \mu(i) \,\}$

$FI = \{\, \langle 0\rangle \,\}$,

$FO = \{\, \langle i-1\rangle' \mid \mu(i) \,\}$

r_{11}: $\{\, \xi \to \xi' \,\}$

$FI = \{\, \xi' \,\}$

$$\langle i-1\rangle' w_\xi [1]$$

Rotation

r_8: $\{\, \xi' \to \xi \,\}$
$FI = \{\, \xi \,\}$

r_9: $\{\, [i]' \to [i] \mid \mu(i) \,\}$
$FO = \{\, [i]' \mid \mu(i) \,\}$

r_{10}: $\{\, \langle i-1 \rangle' \to \langle i-1 \rangle \mid \mu(i) \,\}$
$FI = \{\, \langle 0 \rangle \,\}$,
$FO = \{\, \langle i-1 \rangle' \mid \mu(i) \,\}$

r_{11}: $\{\, \xi \to \xi' \,\}$
$FI = \{\, \xi' \,\}$

$$\langle i-1 \rangle w_\xi [1]$$

Rotation

r_8: $\{\, \xi' \to \xi \,\}$ — $FI = \{\, \xi \,\}$

r_9: $\{\, [i]' \to [i] \mid \mu(i) \,\}$ — $FO = \{\, [i]' \mid \mu(i) \,\}$

r_{10}: $\{\, \langle i-1 \rangle' \to \langle i-1 \rangle \mid \mu(i) \,\}$ — $FI = \{\, \langle 0 \rangle \,\}$, $FO = \{\, \langle i-1 \rangle' \mid \mu(i) \,\}$

r_{11}: $\{\, \xi \to \xi' \,\}$ — $FI = \{\, \xi' \,\}$

$$\langle i-1 \rangle w_\xi [1]$$

Rotation

r_8: $\{\, \xi' \to \xi \,\}$

$FI = \{\, \xi \,\}$

r_9: $\{\, [i]' \to [i] \mid \mu(i) \,\}$

$FO = \{\, [i]' \mid \mu(i) \,\}$

r_{10}: $\{\, \langle i-1\rangle' \to \langle i-1\rangle \mid \mu(i) \,\}$

$FI = \{\, \langle 0\rangle \,\}$,

$FO = \{\, \langle i-1\rangle' \mid \mu(i) \,\}$

r_{11}: $\{\, \xi \to \xi' \,\}$

$FI = \{\, \xi' \,\}$

$$\langle i-1\rangle w_{\xi'}[1]$$

Rotation

r_8: $\{\, \xi' \to \xi \,\}$
$FI = \{\, \xi \,\}$

r_9: $\{\ [i]' \to [i] \mid \mu(i)\ \}$
$FO = \{\ [i]' \mid \mu(i)\ \}$

r_{10}: $\{\ \langle i-1\rangle' \to \langle i-1\rangle \mid \mu(i)\ \}$
$FI = \{\ \langle 0\rangle\ \}$,
$FO = \{\ \langle i-1\rangle' \mid \mu(i)\ \}$

r_{11}: $\{\, \xi \to \xi' \,\}$
$FI = \{\, \xi' \,\}$

r_{12}: $\{\ \langle 0\rangle \xrightarrow{l} \lambda\ \}$
$FI = \{\ \langle i\rangle \mid \mu(i)\ \}$

$$\langle i-1\rangle w_{\xi'}[1]$$

Rotation

r_5: $\{ \xi \to \xi' \}$ — $FI = \{ \xi' \}$

r_6: $\{ [i-1] \to [i]' \mid \mu(i) \}$ — $FO = \{ [i-1] \mid \mu(i) \}$

r_7: $\{ \langle i \rangle \to \langle i-1 \rangle' \mid \mu(i) \}$ — $FO = \{ \langle i \rangle \mid \mu(i) \}$

r_8: $\{ \xi' \to \xi \}$ — $FI = \{ \xi \}$

$$\langle i-1 \rangle w_{\xi'}[1]$$

Rotation

r_5: $\{\, \xi \to \xi' \,\}$ — $FI = \{\, \xi' \,\}$

r_6: $\{\, [i-1] \to [i]' \mid \mu(i) \,\}$ — $FO = \{\, [i-1] \mid \mu(i) \,\}$

r_7: $\{\, \langle i \rangle \to \langle i-1 \rangle' \mid \mu(i) \,\}$ — $FO = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_8: $\{\, \xi' \to \xi \,\}$ — $FI = \{\, \xi \,\}$

$$\langle i-1 \rangle w_\xi [1]$$

Rotation

r_5: $\{\, \xi \to \xi' \,\}$ / $FI = \{\, \xi' \,\}$

r_6: $\{\, [i-1] \to [i]' \mid \mu(i) \,\}$ / $FO = \{\, [i-1] \mid \mu(i) \,\}$

r_7: $\{\, \langle i \rangle \to \langle i-1 \rangle' \mid \mu(i) \,\}$ / $FO = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_8: $\{\, \xi' \to \xi \,\}$ / $FI = \{\, \xi \,\}$

$$\langle i-1 \rangle w_\xi [1]$$

Rotation

r_5: $\{\, \xi \to \xi' \,\}$ — $FI = \{\, \xi' \,\}$

r_6: $\{\, [i-1] \to [i]' \mid \mu(i) \,\}$ — $FO = \{\, [i-1] \mid \mu(i) \,\}$

r_7: $\{\, \langle i \rangle \to \langle i-1 \rangle' \mid \mu(i) \,\}$ — $FO = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_8: $\{\, \xi' \to \xi \,\}$ — $FI = \{\, \xi \,\}$

$$\langle i-2 \rangle' w_\xi [1]$$

Rotation

r_5: $\{\, \xi \to \xi' \,\}$ — $FI = \{\, \xi' \,\}$

r_6: $\{\, [i-1] \to [i]' \mid \mu(i) \,\}$ — $FO = \{\, [i-1] \mid \mu(i) \,\}$

r_7: $\{\, \langle i \rangle \to \langle i-1 \rangle' \mid \mu(i) \,\}$ — $FO = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_8: $\{\, \xi' \to \xi \,\}$ — $FI = \{\, \xi \,\}$

$$\langle i-2 \rangle' w_\xi [1]$$

Rotation

r_5: $\{\, \xi \to \xi' \,\}$; $FI = \{\, \xi' \,\}$

r_6: $\{\, [i-1] \to [i]' \mid \mu(i) \,\}$; $FO = \{\, [i-1] \mid \mu(i) \,\}$

r_7: $\{\, \langle i \rangle \to \langle i-1 \rangle' \mid \mu(i) \,\}$; $FO = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_8: $\{\, \xi' \to \xi \,\}$; $FI = \{\, \xi \,\}$

$$\langle i-2 \rangle' w_{\xi} [2]'$$

Rotation

r_5: $\{\, \xi \to \xi' \,\}$ — $FI = \{\, \xi' \,\}$

r_6: $\{\, [i-1] \to [i]' \mid \mu(i) \,\}$ — $FO = \{\, [i-1] \mid \mu(i) \,\}$

r_7: $\{\, \langle i \rangle \to \langle i-1 \rangle' \mid \mu(i) \,\}$ — $FO = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_8: $\{\, \xi' \to \xi \,\}$ — $FI = \{\, \xi \,\}$

$$\langle i-2 \rangle' w_\xi [2]'$$

Rotation

r_5: $\{\, \xi \to \xi' \,\}$ — $FI = \{\, \xi' \,\}$

r_6: $\{\, [i-1] \to [i]' \mid \mu(i) \,\}$ — $FO = \{\, [i-1] \mid \mu(i) \,\}$

r_7: $\{\, \langle i \rangle \to \langle i-1 \rangle' \mid \mu(i) \,\}$ — $FO = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_8: $\{\, \xi' \to \xi \,\}$ — $FI = \{\, \xi \,\}$

$$\langle i-2 \rangle' w_{\xi'} [2]'$$

Rotation

r_8: $\{\, \xi' \to \xi \,\}$ — $FI = \{\, \xi \,\}$

r_9: $\{\, [i]' \to [i] \mid \mu(i) \,\}$ — $FO = \{\, [i]' \mid \mu(i) \,\}$

r_{10}: $\{\, \langle i-1\rangle' \to \langle i-1\rangle \mid \mu(i) \,\}$ — $FI = \{\, \langle 0\rangle \,\}$, $FO = \{\, \langle i-1\rangle' \mid \mu(i) \,\}$

r_{11}: $\{\, \xi \to \xi' \,\}$ — $FI = \{\, \xi' \,\}$

r_{12}: $\{\, \langle 0\rangle \xrightarrow{l} \lambda \,\}$ — $FI = \{\, \langle i\rangle \mid \mu(i) \,\}$

$$\langle i-2\rangle' w_{\xi'} [2]'$$

Rotation

r_8: $\{\, \xi' \to \xi \,\}$ — $FI = \{\, \xi \,\}$

r_9: $\{\, [i]' \to [i] \mid \mu(i) \,\}$ — $FO = \{\, [i]' \mid \mu(i) \,\}$

r_{10}: $\{\, \langle i-1\rangle' \to \langle i-1\rangle \mid \mu(i) \,\}$ — $FI = \{\, \langle 0\rangle \,\}$, $FO = \{\, \langle i-1\rangle' \mid \mu(i) \,\}$

r_{11}: $\{\, \xi \to \xi' \,\}$ — $FI = \{\, \xi' \,\}$

r_{12}: $\{\, \langle 0\rangle \xrightarrow{l} \lambda \,\}$ — $FI = \{\, \langle i\rangle \mid \mu(i) \,\}$

$$\langle i-2\rangle w_\xi [2]$$

Rotation

r_8: $\{\, \xi' \to \xi \,\}$; $FI = \{\, \xi \,\}$

r_9: $\{\, [i]' \to [i] \mid \mu(i) \,\}$; $FO = \{\, [i]' \mid \mu(i) \,\}$

r_{10}: $\{\, \langle i-1 \rangle' \to \langle i-1 \rangle \mid \mu(i) \,\}$; $FI = \{\, \langle 0 \rangle \,\}$, $FO = \{\, \langle i-1 \rangle' \mid \mu(i) \,\}$

r_{11}: $\{\, \xi \to \xi' \,\}$; $FI = \{\, \xi' \,\}$

r_{12}: $\{\, \langle 0 \rangle \xrightarrow{l} \lambda \,\}$; $FI = \{\, \langle i \rangle \mid \mu(i) \,\}$

$$\langle 0 \rangle w_{\xi'} [i]$$

Rotation

r_{11}: $\{\, \xi \to \xi' \,\}$; $FI = \{\, \xi' \,\}$

r_{12}: $\{\, \langle 0 \rangle \stackrel{l}{\to} \lambda \,\}$; $FI = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_{13}: $\{\, [i] \to x_i \mid \mu(i) \,\}$; $FI = \{\, \langle 0 \rangle \,\}$, $FO = \{\, [i] \mid \mu(i) \,\}$

r_{14}: $\{\, \xi' \to \xi \,\}$; $FI = \{\, \xi \,\}$

$$\langle 0 \rangle w_{\xi'}[i]$$

Rotation

r_{11}: $\{\, \xi \to \xi' \,\}$

$FI = \{\, \xi' \,\}$

r_{12}: $\{\, \langle 0 \rangle \xrightarrow{l} \lambda \,\}$

$FI = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_{13}: $\{\, [i] \to x_i \mid \mu(i) \,\}$

$FI = \{\, \langle 0 \rangle \,\}$,

$FO = \{\, [i] \mid \mu(i) \,\}$

r_{14}: $\{\, \xi' \to \xi \,\}$

$FI = \{\, \xi \,\}$

$w_{\xi'}[i]$

Rotation

r_{11}: $\{\, \xi \to \xi' \,\}$ — $FI = \{\, \xi' \,\}$

r_{12}: $\{\, \langle 0 \rangle \xrightarrow{l} \lambda \,\}$ — $FI = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_{13}: $\{\, [i] \to x_i \mid \mu(i) \,\}$ — $FI = \{\, \langle 0 \rangle \,\}$, $FO = \{\, [i] \mid \mu(i) \,\}$

r_{14}: $\{\, \xi' \to \xi \,\}$ — $FI = \{\, \xi \,\}$

$w_{\xi'}[i]$

Rotation

r_{11}: $\{\, \xi \to \xi' \,\}$

$FI = \{\, \xi' \,\}$

r_{12}: $\{\, \langle 0 \rangle \xrightarrow{l} \lambda \,\}$

$FI = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_{13}: $\{\, [i] \to x_i \mid \mu(i) \,\}$

$FI = \{\, \langle 0 \rangle \,\}$,
$FO = \{\, [i] \mid \mu(i) \,\}$

r_{14}: $\{\, \xi' \to \xi \,\}$

$FI = \{\, \xi \,\}$

$w_{\xi'} x_i$

Rotation

r_{11}: $\{\, \xi \to \xi' \,\}$; $FI = \{\, \xi' \,\}$

r_{12}: $\{\, \langle 0 \rangle \xrightarrow{l} \lambda \,\}$; $FI = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_{13}: $\{\, [i] \to x_i \mid \mu(i) \,\}$; $FI = \{\, \langle 0 \rangle \,\}$, $FO = \{\, [i] \mid \mu(i) \,\}$

r_{14}: $\{\, \xi' \to \xi \,\}$; $FI = \{\, \xi \,\}$

$w_{\xi'} x_i$

Rotation

r_{11}: $\{\, \xi \to \xi' \,\}$ — $FI = \{\, \xi' \,\}$

r_{12}: $\{\, \langle 0 \rangle \xrightarrow{l} \lambda \,\}$ — $FI = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_{13}: $\{\, [i] \to x_i \mid \mu(i) \,\}$ — $FI = \{\, \langle 0 \rangle \,\}$, $FO = \{\, [i] \mid \mu(i) \,\}$

r_{14}: $\{\, \xi' \to \xi \,\}$ — $FI = \{\, \xi \,\}$

$w_\xi x_i$

Checking for the End

r_{14}: $\{\, \xi' \to \xi \,\}$

$FI = \{\, \xi \,\}$

e_1: $\{\, S \to S' \,\}$

$PI = \{\, S \,\}$

e_2: $\emptyset$

$PI = \{\, S' \,\}$,

$FI = W \setminus \{\, S', \#, \xi \,\}$

$w_\xi x_i$

Checking for the End

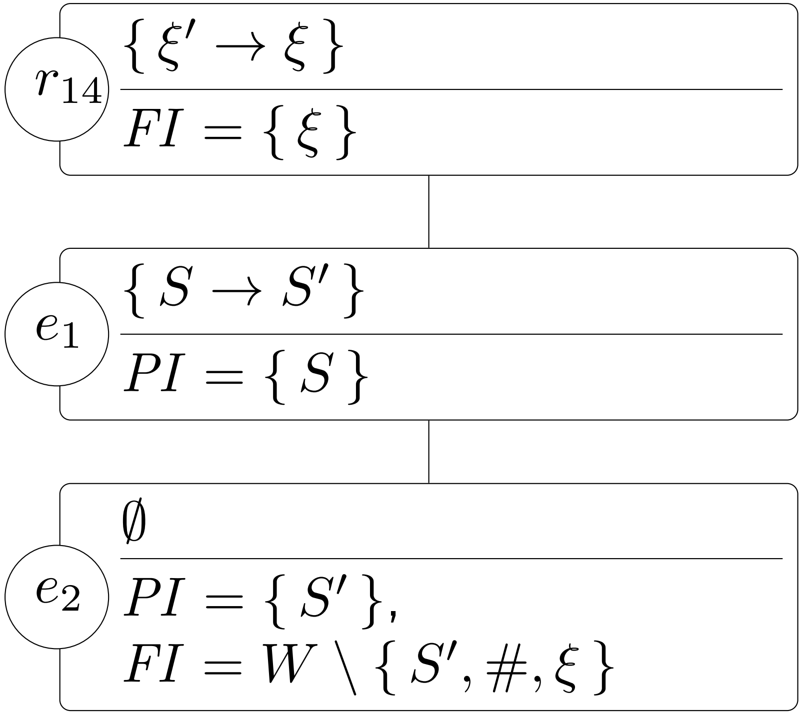
r_{14}: $\{\, \xi' \to \xi \,\}$ — $FI = \{\, \xi \,\}$

e_1: $\{\, S \to S' \,\}$ — $PI = \{\, S \,\}$

e_2: $\emptyset$ — $PI = \{\, S' \,\}$, $FI = W \setminus \{\, S', \#, \xi \,\}$

$u_\xi S v_\xi$

Checking for the End



$u_\xi S' v_\xi$

Rotation (continued)

r_{11}: $\{\,\xi \to \xi'\,\}$; $FI = \{\,\xi'\,\}$

r_{12}: $\{\,\langle 0\rangle \xrightarrow{l} \lambda\,\}$; $FI = \{\,\langle i\rangle \mid \mu(i)\,\}$

r_{13}: $\{\,[i] \to x_i \mid \mu(i)\,\}$; $FI = \{\,\langle 0\rangle\,\}$, $FO = \{\,[i] \mid \mu(i)\,\}$

r_{14}: $\{\,\xi' \to \xi\,\}$; $FI = \{\,\xi\,\}$

$w_\xi x_i$

Rotation (continued)

r_{11}: $\{\, \xi \to \xi' \,\}$ — $FI = \{\, \xi' \,\}$

r_{12}: $\{\, \langle 0 \rangle \xrightarrow{l} \lambda \,\}$ — $FI = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_{13}: $\{\, [i] \to x_i \mid \mu(i) \,\}$ — $FI = \{\, \langle 0 \rangle \,\}$, $FO = \{\, [i] \mid \mu(i) \,\}$

r_{14}: $\{\, \xi' \to \xi \,\}$ — $FI = \{\, \xi \,\}$

$w_\xi x_i$

Rotation (continued)

r_{11}: $\{\, \xi \to \xi' \,\}$ — $FI = \{\, \xi' \,\}$

r_{12}: $\{\, \langle 0 \rangle \xrightarrow{l} \lambda \,\}$ — $FI = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_{13}: $\{\, [i] \to x_i \mid \mu(i) \,\}$ — $FI = \{\, \langle 0 \rangle \,\}$, $FO = \{\, [i] \mid \mu(i) \,\}$

r_{14}: $\{\, \xi' \to \xi \,\}$ — $FI = \{\, \xi \,\}$

$w_{\xi'} x_i$

Rotation (continued)

r_8: $\{\, \xi' \to \xi \,\}$; $FI = \{\, \xi \,\}$

r_9: $\{\, [i]' \to [i] \mid \mu(i) \,\}$; $FO = \{\, [i]' \mid \mu(i) \,\}$

r_{10}: $\{\, \langle i-1\rangle' \to \langle i-1\rangle \mid \mu(i) \,\}$; $FI = \{\, \langle 0\rangle \,\}$, $FO = \{\, \langle i-1\rangle' \mid \mu(i) \,\}$

r_{11}: $\{\, \xi \to \xi' \,\}$; $FI = \{\, \xi' \,\}$

$w_{\xi'} x_i$

Rotation (continued)

r_8: $\{ \xi' \to \xi \}$ — $FI = \{ \xi \}$

r_9: $\{ [i]' \to [i] \mid \mu(i) \}$ — $FO = \{ [i]' \mid \mu(i) \}$

r_{10}: $\{ \langle i-1 \rangle' \to \langle i-1 \rangle \mid \mu(i) \}$ — $FI = \{ \langle 0 \rangle \}$, $FO = \{ \langle i-1 \rangle' \mid \mu(i) \}$

r_{11}: $\{ \xi \to \xi' \}$ — $FI = \{ \xi' \}$

$$w_{\xi'} x_i$$

Rotation (continued)

r_8: $\{\,\xi' \to \xi\,\}$ — $FI = \{\,\xi\,\}$

r_9: $\{\,[i]' \to [i] \mid \mu(i)\,\}$ — $FO = \{\,[i]' \mid \mu(i)\,\}$

r_{10}: $\{\,\langle i-1\rangle' \to \langle i-1\rangle \mid \mu(i)\,\}$ — $FI = \{\,\langle 0\rangle\,\}$, $FO = \{\,\langle i-1\rangle' \mid \mu(i)\,\}$

r_{11}: $\{\,\xi \to \xi'\,\}$ — $FI = \{\,\xi'\,\}$

$w_\xi x_i$

Rotation (continued)

r_5: $\{\, \xi \to \xi' \,\}$ — $FI = \{\, \xi' \,\}$

r_6: $\{\, [i-1] \to [i]' \mid \mu(i) \,\}$ — $FO = \{\, [i-1] \mid \mu(i) \,\}$

r_7: $\{\, \langle i \rangle \to \langle i-1 \rangle' \mid \mu(i) \,\}$ — $FO = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_8: $\{\, \xi' \to \xi \,\}$ — $FI = \{\, \xi \,\}$

$$w_\xi x_i$$

Rotation (continued)

r_5: $\{ \xi \to \xi' \}$ — $FI = \{ \xi' \}$

r_6: $\{ [i-1] \to [i]' \mid \mu(i) \}$ — $FO = \{ [i-1] \mid \mu(i) \}$

r_7: $\{ \langle i \rangle \to \langle i-1 \rangle' \mid \mu(i) \}$ — $FO = \{ \langle i \rangle \mid \mu(i) \}$

r_8: $\{ \xi' \to \xi \}$ — $FI = \{ \xi \}$

$w_\xi x_i$

Rotation (continued)

r_5: $\{\, \xi \to \xi' \,\}$ — $FI = \{\, \xi' \,\}$

r_6: $\{\, [i-1] \to [i]' \mid \mu(i) \,\}$ — $FO = \{\, [i-1] \mid \mu(i) \,\}$

r_7: $\{\, \langle i \rangle \to \langle i-1 \rangle' \mid \mu(i) \,\}$ — $FO = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_8: $\{\, \xi' \to \xi \,\}$ — $FI = \{\, \xi \,\}$

$$w_{\xi'} x_i$$

Rotation (continued)

r_1: $\{\, [0] \xrightarrow{r} \lambda \,\}$ — $PI = \{\, \xi, \xi' \,\}$

r_2: $\{\, \lambda \xrightarrow{r} [0] \,\}$ — $FI = \{\, [0] \,\}$

r_3: $\{\, \langle i \rangle \to x_i \mid \mu(i) \,\}$ — $FO = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_4: $\{\, x_i \to \langle i \rangle \mid \mu(i) \,\}$ — $FI = PO = \{\langle i \rangle, \langle i \rangle' \mid \mu(i)\}$

r_5: $\{\, \xi \to \xi' \,\}$ — $FI = \{\, \xi' \,\}$

$w_{\xi'} x_i$

Rotation (continued)

r_1: $\{ [0] \xrightarrow{r} \lambda \}$ — $PI = \{ \xi, \xi' \}$

r_2: $\{ \lambda \xrightarrow{r} [0] \}$ — $FI = \{ [0] \}$

r_3: $\{ \langle i \rangle \to x_i \mid \mu(i) \}$ — $FO = \{ \langle i \rangle \mid \mu(i) \}$

r_4: $\{ x_i \to \langle i \rangle \mid \mu(i) \}$ — $FI = PO = \{ \langle i \rangle, \langle i \rangle' \mid \mu(i) \}$

r_5: $\{ \xi \to \xi' \}$ — $FI = \{ \xi' \}$

$w_{\xi'} x_i$

Rotation (continued)

r_1: $\{ [0] \xrightarrow{r} \lambda \}$ — $PI = \{ \xi, \xi' \}$

r_2: $\{ \lambda \xrightarrow{r} [0] \}$ — $FI = \{ [0] \}$

r_3: $\{ \langle i \rangle \to x_i \mid \mu(i) \}$ — $FO = \{ \langle i \rangle \mid \mu(i) \}$

r_4: $\{ x_i \to \langle i \rangle \mid \mu(i) \}$ — $FI = PO = \{ \langle i \rangle, \langle i \rangle' \mid \mu(i) \}$

r_5: $\{ \xi \to \xi' \}$ — $FI = \{ \xi' \}$

$$w_{\xi'} x_i [0]$$

Rotation (continued)

r_1: $\{\, [0] \xrightarrow{r} \lambda \,\}$ — $PI = \{\, \xi, \xi' \,\}$

r_2: $\{\, \lambda \xrightarrow{r} [0] \,\}$ — $FI = \{\, [0] \,\}$

r_3: $\{\, \langle i \rangle \to x_i \mid \mu(i) \,\}$ — $FO = \{\, \langle i \rangle \mid \mu(i) \,\}$

r_4: $\{\, x_i \to \langle i \rangle \mid \mu(i) \,\}$ — $FI = PO = \{ \langle i \rangle, \langle i \rangle' \mid \mu(i) \}$

r_5: $\{\, \xi \to \xi' \,\}$ — $FI = \{\, \xi' \,\}$

$$w_{\xi'} x_i [0]$$

Rotation (continued)

r_1: $\{ [0] \xrightarrow{r} \lambda \}$, $PI = \{ \xi, \xi' \}$

r_2: $\{ \lambda \xrightarrow{r} [0] \}$, $FI = \{ [0] \}$

r_3: $\{ \langle i \rangle \to x_i \mid \mu(i) \}$, $FO = \{ \langle i \rangle \mid \mu(i) \}$

r_4: $\{ x_i \to \langle i \rangle \mid \mu(i) \}$, $FI = PO = \{ \langle i \rangle, \langle i \rangle' \mid \mu(i) \}$

r_5: $\{ \xi \to \xi' \}$, $FI = \{ \xi' \}$

$$w_{\xi'} x_i$$

Simulation of Rules $A \to \lambda$

λ_1: $\{\, \lambda \to \xi \,\}$

$FI = \{\, \xi \,\}$

λ_2: $\{\, \xi \to A \mid A \to \lambda \in P \,\} \cup \{\, \xi \to \xi \,\}$

$\emptyset$

λ_3: $\{\, \xi' \to \lambda \,\}$

$\emptyset$

$w_{\xi'} x_i$

Simulation of Rules $A \to \lambda$

λ_1: $\{\, \lambda \to \xi \,\}$; $FI = \{\, \xi \,\}$

λ_2: $\{\, \xi \to A \mid A \to \lambda \in P \,\} \cup \{\, \xi \to \xi \,\}$; $\emptyset$

λ_3: $\{\, \xi' \to \lambda \,\}$; $\emptyset$

$u_{\xi'}$

Simulation of Rules $A \to \lambda$

λ_1: $\{ \lambda \to \xi \}$

$FI = \{ \xi \}$

λ_2: $\{ \xi \to A \mid A \to \lambda \in P \} \cup \{ \xi \to \xi \}$

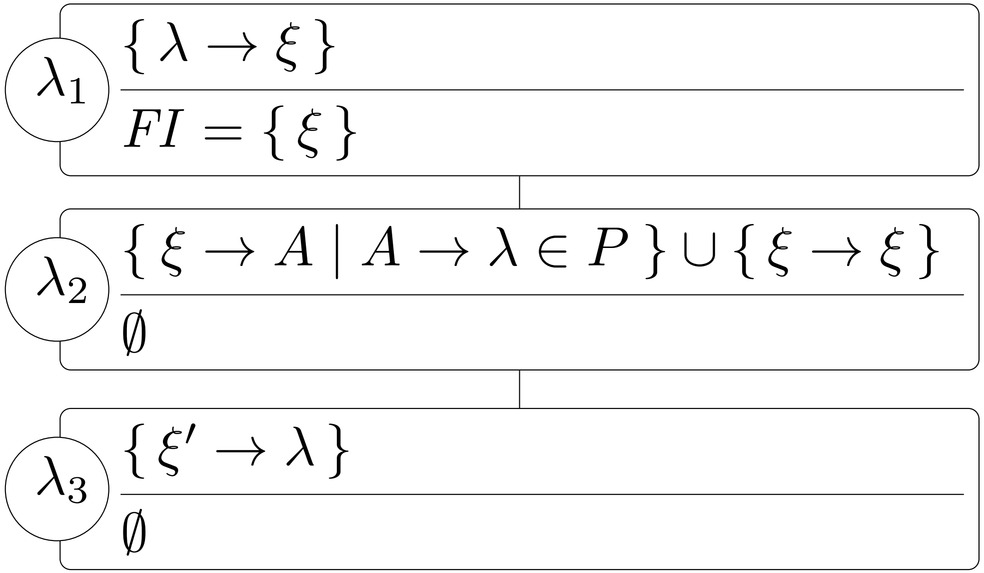
$\emptyset$

λ_3: $\{ \xi' \to \lambda \}$

$\emptyset$

$u_{\xi'\xi}$

Simulation of Rules $A \to \lambda$



$u_{\xi'\xi}$

Simulation of Rules $A \to \lambda$

λ_1: $\{\,\lambda \to \xi\,\}$ — $FI = \{\,\xi\,\}$

λ_2: $\{\,\xi \to A \mid A \to \lambda \in P\,\} \cup \{\,\xi \to \xi\,\}$ — $\emptyset$

λ_3: $\{\,\xi' \to \lambda\,\}$ — $\emptyset$

$u_{\xi'\xi},\ u'_{\xi'}$

Simulation of Rules $A \to \lambda$

λ_1: $\{\, \lambda \to \xi \,\}$; $FI = \{\, \xi \,\}$

λ_2: $\{\, \xi \to A \mid A \to \lambda \in P \,\} \cup \{\, \xi \to \xi \,\}$; $\emptyset$

λ_3: $\{\, \xi' \to \lambda \,\}$; $\emptyset$

$u_{\xi'\xi},\ u'_{\xi'}$

Simulation of Rules $A \to \lambda$

λ_1: $\{\, \lambda \to \xi \,\}$ — $FI = \{\, \xi \,\}$

λ_2: $\{\, \xi \to A \mid A \to \lambda \in P \,\} \cup \{\, \xi \to \xi \,\}$ — $\emptyset$

λ_3: $\{\, \xi' \to \lambda \,\}$ — $\emptyset$

$u_\xi,\ u'$

Simulation of Rules $A \to \lambda$

λ_1: $\{\, \lambda \to \xi \,\}$ — $FI = \{\, \xi \,\}$

λ_2: $\{\, \xi \to A \mid A \to \lambda \in P \,\} \cup \{\, \xi \to \xi \,\}$ — $\emptyset$

λ_3: $\{\, \xi' \to \lambda \,\}$ — $\emptyset$

χ: $\{\, x \to A \mid \psi(A, x) \,\} \cup \{\, \# \to \# \,\}$ — $PI = \{\, \xi, \xi' \,\}$

$u_\xi,\ u'$

Simulation of Rules $A \to \lambda$

λ_1: $\{ \lambda \to \xi \}$ — $FI = \{ \xi \}$

λ_2: $\{ \xi \to A \mid A \to \lambda \in P \} \cup \{ \xi \to \xi \}$ — $\emptyset$

λ_3: $\{ \xi' \to \lambda \}$ — $\emptyset$

χ: $\{ x \to A \mid \psi(A,x) \} \cup \{ \# \to \# \}$ — $PI = \{ \xi, \xi' \}$

u_ξ, u'

Simulation of Rules $A \to \lambda$

λ_1: $\{\, \lambda \to \xi \,\}$ / $FI = \{\, \xi \,\}$

λ_2: $\{\, \xi \to A \mid A \to \lambda \in P \,\} \cup \{\, \xi \to \xi \,\}$ / $\emptyset$

λ_3: $\{\, \xi' \to \lambda \,\}$ / $\emptyset$

χ: $\{\, x \to A \mid \psi(A, x) \,\} \cup \{\, \# \to \# \,\}$ / $PI = \{\, \xi, \xi' \,\}$

u'

Simulation of Rules $A \to \lambda$

λ_1: $\{\, \lambda \to \xi \,\}$; $FI = \{\, \xi \,\}$

λ_2: $\{\, \xi \to A \mid A \to \lambda \in P \,\} \cup \{\, \xi \to \xi \,\}$; $\emptyset$

λ_3: $\{\, \xi' \to \lambda \,\}$; $\emptyset$

χ: $\{\, x \to A \mid \psi(A,x) \,\} \cup \{\, \# \to \# \,\}$; $PI = \{\, \xi, \xi' \,\}$

u'_ξ

Simulation of Rules $A \to \lambda$

λ_1: $\{\, \lambda \to \xi \,\}$ — $FI = \{\, \xi \,\}$

λ_2: $\{\, \xi \to A \mid A \to \lambda \in P \,\} \cup \{\, \xi \to \xi \,\}$ — $\emptyset$

λ_3: $\{\, \xi' \to \lambda \,\}$ — $\emptyset$

χ: $\{\, x \to A \mid \psi(A, x) \,\} \cup \{\, \# \to \# \,\}$ — $PI = \{\, \xi, \xi' \,\}$

w_ξ

Simulation of Rules $A \to \lambda$

Node	Rules	Filter
λ_1	$\{\, \lambda \to \xi \,\}$	$FI = \{\, \xi \,\}$
λ_2	$\{\, \xi \to A \mid A \to \lambda \in P \,\} \cup \{\, \xi \to \xi \,\}$	$\emptyset$
λ_3	$\{\, \xi' \to \lambda \,\}$	$\emptyset$
χ	$\{\, x \to A \mid \psi(A, x) \,\} \cup \{\, \# \to \# \,\}$	$PI = \{\, \xi, \xi' \,\}$

w_ξ

Simulation of Rules $A \to \lambda$

λ_1: $\{\, \lambda \to \xi \,\}$ — $FI = \{\, \xi \,\}$

λ_2: $\{\, \xi \to A \mid A \to \lambda \in P \,\} \cup \{\, \xi \to \xi \,\}$ — $\emptyset$

λ_3: $\{\, \xi' \to \lambda \,\}$ — $\emptyset$

χ: $\{\, x \to A \mid \psi(A, x) \,\} \cup \{\, \# \to \# \,\}$ — $PI = \{\, \xi, \xi' \,\}$

$w_\xi,\ w'$

Simulation of Rules $A \to \lambda$

λ_1: $\{\, \lambda \to \xi \,\}$ / $FI = \{\, \xi \,\}$

λ_2: $\{\, \xi \to A \mid A \to \lambda \in P \,\} \cup \{\, \xi \to \xi \,\}$ / $\emptyset$

λ_3: $\{\, \xi' \to \lambda \,\}$ / $\emptyset$

χ: $\{\, x \to A \mid \psi(A, x) \,\} \cup \{\, \# \to \# \,\}$ / $PI = \{\, \xi, \xi' \,\}$

w_ξ, w'

Simulation of Rules $A \to x$

λ_1: $\{\,\lambda \to \xi\,\}$ — $FI = \{\,\xi\,\}$

λ_2: $\{\,\xi \to A \mid A \to \lambda \in P\,\} \cup \{\,\xi \to \xi\,\}$ — $\emptyset$

λ_3: $\{\,\xi' \to \lambda\,\}$ — $\emptyset$

χ: $\{\,x \to A \mid \psi(A,x)\,\} \cup \{\,\# \to \#\,\}$ — $PI = \{\,\xi, \xi'\,\}$

w_ξ

Simulation of Rules $A \to x$

λ_1: $\{ \lambda \to \xi \}$ / $FI = \{ \xi \}$

λ_2: $\{ \xi \to A \mid A \to \lambda \in P \} \cup \{ \xi \to \xi \}$ / $\emptyset$

λ_3: $\{ \xi' \to \lambda \}$ / $\emptyset$

χ: $\{ x \to A \mid \psi(A, x) \} \cup \{ \# \to \# \}$ / $PI = \{ \xi, \xi' \}$

σ_1: $\{ \xi \to \xi' \}$ / $FI = \{ \xi' \}$

w_ξ

Simulation of Rules $A \to x$

λ_1: $\{ \lambda \to \xi \}$ — $FI = \{ \xi \}$

λ_2: $\{ \xi \to A \mid A \to \lambda \in P \} \cup \{ \xi \to \xi \}$ — $\emptyset$

λ_3: $\{ \xi' \to \lambda \}$ — $\emptyset$

χ: $\{ x \to A \mid \psi(A, x) \} \cup \{ \# \to \# \}$ — $PI = \{ \xi, \xi' \}$

σ_1: $\{ \xi \to \xi' \}$ — $FI = \{ \xi' \}$

w_ξ

Simulation of Rules $A \to x$

λ_1: $\{ \lambda \to \xi \}$; $FI = \{ \xi \}$

λ_2: $\{ \xi \to A \mid A \to \lambda \in P \} \cup \{ \xi \to \xi \}$; $\emptyset$

λ_3: $\{ \xi' \to \lambda \}$; $\emptyset$

χ: $\{ x \to A \mid \psi(A, x) \} \cup \{ \# \to \# \}$; $PI = \{ \xi, \xi' \}$

σ_1: $\{ \xi \to \xi' \}$; $FI = \{ \xi' \}$

$w_{\xi'}$

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $Du_{\xi'}\#v_{\xi'}C \quad \to Bu_{\xi'}\#v_{\xi'}A$

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $Du_{\xi'}\#v_{\xi'}C \quad \to Bu_{\xi'}\#v_{\xi'}A$

- mark $r_2(i)$ by $\langle i, i \rangle$ (hopefully at the left end)

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $\langle i,i\rangle u_{\xi'}\#v_{\xi'}C \quad \to Bu_{\xi'}\#v_{\xi'}A$

- mark $r_2(i)$ by $\langle i,i\rangle$ (hopefully at the left end)

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $\langle i, i \rangle u_{\xi'} \# v_{\xi'} C \quad \to B u_{\xi'} \# v_{\xi'} A$

- mark $r_2(i)$ by $\langle i, i \rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j, j]$ (hopefully at the right end)

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $\langle i,i\rangle\, u_{\xi'}\#v_{\xi'}[j,j] \to Bu_{\xi'}\#v_{\xi'}A$

- mark $r_2(i)$ by $\langle i,i\rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j,j]$ (hopefully at the right end)

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $\langle i, i \rangle u_{\xi'} \# v_{\xi'} [j, j] \to B u_{\xi'} \# v_{\xi'} A$

- mark $r_2(i)$ by $\langle i, i \rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j, j]$ (hopefully at the right end)
- decrement alternatingly $\langle i, i-k \rangle$ and $[j, j-k]$ until $\langle i, 0 \rangle$ or $[j, 0]$

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $\langle i, 0\rangle u_{\xi'}\#v_{\xi'}[i, 0] \to Bu_{\xi'}\#v_{\xi'}A$

- mark $r_2(i)$ by $\langle i, i\rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j, j]$ (hopefully at the right end)
- decrement alternatingly $\langle i, i - k\rangle$ and $[j, j - k]$ until $\langle i, 0\rangle$ or $[j, 0]$

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $\langle i, 0\rangle u_{\xi'}\#v_{\xi'}[i, 0] \to Bu_{\xi'}\#v_{\xi'}A$

- mark $r_2(i)$ by $\langle i, i\rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j, j]$ (hopefully at the right end)
- decrement alternatingly $\langle i, i - k\rangle$ and $[j, j - k]$ until $\langle i, 0\rangle$ or $[j, 0]$
- delete $\langle i, 0\rangle$ from the left end

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $u_{\xi'}\#v_{\xi'}[i,0] \to Bu_{\xi'}\#v_{\xi'}A$

- mark $r_2(i)$ by $\langle i,i \rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j,j]$ (hopefully at the right end)
- decrement alternatingly $\langle i,i-k \rangle$ and $[j,j-k]$ until $\langle i,0 \rangle$ or $[j,0]$
- delete $\langle i,0 \rangle$ from the left end

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $u_{\xi'}\#v_{\xi'}[i,0] \to Bu_{\xi'}\#v_{\xi'}A$

- mark $r_2(i)$ by $\langle i,i \rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j,j]$ (hopefully at the right end)
- decrement alternatingly $\langle i,i-k \rangle$ and $[j,j-k]$ until $\langle i,0 \rangle$ or $[j,0]$
- delete $\langle i,0 \rangle$ from the left end
- insert $\langle 0,0 \rangle$ at the left end

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $\langle 0, 0 \rangle u_{\xi'} \# v_{\xi'} [i, 0] \to B u_{\xi'} \# v_{\xi'} A$

- mark $r_2(i)$ by $\langle i, i \rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j, j]$ (hopefully at the right end)
- decrement alternatingly $\langle i, i - k \rangle$ and $[j, j - k]$ until $\langle i, 0 \rangle$ or $[j, 0]$
- delete $\langle i, 0 \rangle$ from the left end
- insert $\langle 0, 0 \rangle$ at the left end

Simulation of Rules $AB \rightarrow CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $\langle 0, 0\rangle u_{\xi'}\#v_{\xi'}[i, 0] \rightarrow Bu_{\xi'}\#v_{\xi'}A$

- mark $r_2(i)$ by $\langle i, i\rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j, j]$ (hopefully at the right end)
- decrement alternatingly $\langle i, i - k\rangle$ and $[j, j - k]$ until $\langle i, 0\rangle$ or $[j, 0]$
- delete $\langle i, 0\rangle$ from the left end
- insert $\langle 0, 0\rangle$ at the left end
- increment alternatingly $\langle k, 0\rangle$ and $[i, k]$ until $\langle i, 0\rangle$ and $[i, i]$

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $\langle i,0\rangle u_{\xi'}\#v_{\xi'}[i,i] \to Bu_{\xi'}\#v_{\xi'}A$

- mark $r_2(i)$ by $\langle i,i\rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j,j]$ (hopefully at the right end)
- decrement alternatingly $\langle i,i-k\rangle$ and $[j,j-k]$ until $\langle i,0\rangle$ or $[j,0]$
- delete $\langle i,0\rangle$ from the left end
- insert $\langle 0,0\rangle$ at the left end
- increment alternatingly $\langle k,0\rangle$ and $[i,k]$ until $\langle i,0\rangle$ and $[i,i]$

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $\langle i, 0 \rangle u_{\xi'} \# v_{\xi'} [i, i] \to B u_{\xi'} \# v_{\xi'} A$

- mark $r_2(i)$ by $\langle i, i \rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j, j]$ (hopefully at the right end)
- decrement alternatingly $\langle i, i-k \rangle$ and $[j, j-k]$ until $\langle i, 0 \rangle$ or $[j, 0]$
- delete $\langle i, 0 \rangle$ from the left end
- insert $\langle 0, 0 \rangle$ at the left end
- increment alternatingly $\langle k, 0 \rangle$ and $[i, k]$ until $\langle i, 0 \rangle$ and $[i, i]$
- delete $[i, i]$ from the right end

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $\langle i, 0 \rangle u_{\xi'} \# v_{\xi'} \quad \to Bu_{\xi'} \# v_{\xi'} A$

- mark $r_2(i)$ by $\langle i, i \rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j, j]$ (hopefully at the right end)
- decrement alternatingly $\langle i, i-k \rangle$ and $[j, j-k]$ until $\langle i, 0 \rangle$ or $[j, 0]$
- delete $\langle i, 0 \rangle$ from the left end
- insert $\langle 0, 0 \rangle$ at the left end
- increment alternatingly $\langle k, 0 \rangle$ and $[i, k]$ until $\langle i, 0 \rangle$ and $[i, i]$
- delete $[i, i]$ from the right end

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $\langle i, 0\rangle u_{\xi'}\#v_{\xi'}[0, 0] \to Bu_{\xi'}\#v_{\xi'}A$

- mark $r_2(i)$ by $\langle i, i\rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j, j]$ (hopefully at the right end)
- decrement alternatingly $\langle i, i - k\rangle$ and $[j, j - k]$ until $\langle i, 0\rangle$ or $[j, 0]$
- delete $\langle i, 0\rangle$ from the left end
- insert $\langle 0, 0\rangle$ at the left end
- increment alternatingly $\langle k, 0\rangle$ and $[i, k]$ until $\langle i, 0\rangle$ and $[i, i]$
- delete $[i, i]$ from the right end
- insert $[0, 0]$ at the right end

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $\langle i, 0\rangle u_{\xi'}\#v_{\xi'}[0, 0] \to Bu_{\xi'}\#v_{\xi'}A$

- mark $r_2(i)$ by $\langle i, i\rangle$ (hopefully at the left end)
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- decrement alternatingly $\langle i, i - k\rangle$ and $[j, j - k]$ until $\langle i, 0\rangle$ or $[j, 0]$
- delete $\langle i, 0\rangle$ from the left end
- insert $\langle 0, 0\rangle$ at the left end
- increment alternatingly $\langle k, 0\rangle$ and $[i, k]$ until $\langle i, 0\rangle$ and $[i, i]$
- delete $[i, i]$ from the right end
- insert $[0, 0]$ at the right end
- increment alternatingly $\langle i, k\rangle$ and $[k, k]$ until $\langle i, i\rangle$ and $[i, i]$

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $\langle i,i\rangle u_{\xi'}\#v_{\xi'}[i,i] \to Bu_{\xi'}\#v_{\xi'}A$

- mark $r_2(i)$ by $\langle i,i\rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j,j]$ (hopefully at the right end)
- decrement alternatingly $\langle i,i-k\rangle$ and $[j,j-k]$ until $\langle i,0\rangle$ or $[j,0]$
- delete $\langle i,0\rangle$ from the left end
- insert $\langle 0,0\rangle$ at the left end
- increment alternatingly $\langle k,0\rangle$ and $[i,k]$ until $\langle i,0\rangle$ and $[i,i]$
- delete $[i,i]$ from the right end
- insert $[0,0]$ at the right end
- increment alternatingly $\langle i,k\rangle$ and $[k,k]$ until $\langle i,i\rangle$ and $[i,i]$

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $\langle i,i\rangle u_{\xi'}\#v_{\xi'}[i,i] \to Bu_{\xi'}\#v_{\xi'}A$

- mark $r_2(i)$ by $\langle i,i\rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j,j]$ (hopefully at the right end)
- decrement alternatingly $\langle i,i-k\rangle$ and $[j,j-k]$ until $\langle i,0\rangle$ or $[j,0]$
- delete $\langle i,0\rangle$ from the left end
- insert $\langle 0,0\rangle$ at the left end
- increment alternatingly $\langle k,0\rangle$ and $[i,k]$ until $\langle i,0\rangle$ and $[i,i]$
- delete $[i,i]$ from the right end
- insert $[0,0]$ at the right end
- increment alternatingly $\langle i,k\rangle$ and $[k,k]$ until $\langle i,i\rangle$ and $[i,i]$
- replace $[i,i]$ by $l_1(i)$

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $\langle i,i\rangle u_{\xi'}\#v_{\xi'}A \quad \to Bu_{\xi'}\#v_{\xi'}A$

- mark $r_2(i)$ by $\langle i,i\rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j,j]$ (hopefully at the right end)
- decrement alternatingly $\langle i,i-k\rangle$ and $[j,j-k]$ until $\langle i,0\rangle$ or $[j,0]$
- delete $\langle i,0\rangle$ from the left end
- insert $\langle 0,0\rangle$ at the left end
- increment alternatingly $\langle k,0\rangle$ and $[i,k]$ until $\langle i,0\rangle$ and $[i,i]$
- delete $[i,i]$ from the right end
- insert $[0,0]$ at the right end
- increment alternatingly $\langle i,k\rangle$ and $[k,k]$ until $\langle i,i\rangle$ and $[i,i]$
- replace $[i,i]$ by $l_1(i)$

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $\langle i, i \rangle u_{\xi'} \# v_{\xi'} A \quad \to B u_{\xi'} \# v_{\xi'} A$

- mark $r_2(i)$ by $\langle i, i \rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j, j]$ (hopefully at the right end)
- decrement alternatingly $\langle i, i-k \rangle$ and $[j, j-k]$ until $\langle i, 0 \rangle$ or $[j, 0]$
- delete $\langle i, 0 \rangle$ from the left end
- insert $\langle 0, 0 \rangle$ at the left end
- increment alternatingly $\langle k, 0 \rangle$ and $[i, k]$ until $\langle i, 0 \rangle$ and $[i, i]$
- delete $[i, i]$ from the right end
- insert $[0, 0]$ at the right end
- increment alternatingly $\langle i, k \rangle$ and $[k, k]$ until $\langle i, i \rangle$ and $[i, i]$
- replace $[i, i]$ by $l_1(i)$
- replace $\langle i, i \rangle$ by $l_2(i)$

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $B\,u_{\xi'}\#v_{\xi'}A \quad \to Bu_{\xi'}\#v_{\xi'}A$

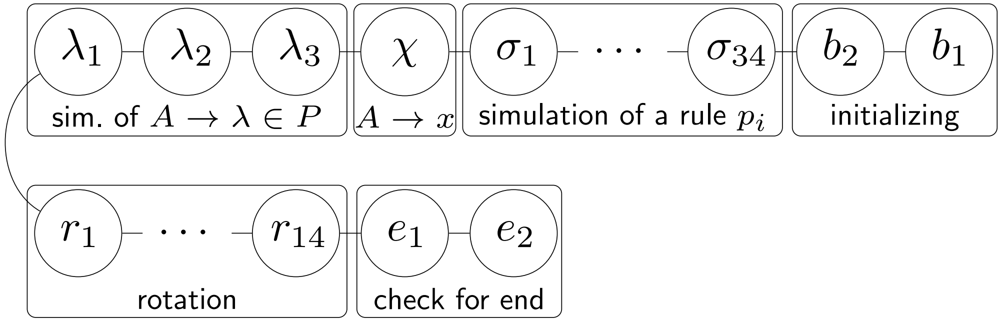
- mark $r_2(i)$ by $\langle i, i \rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j, j]$ (hopefully at the right end)
- decrement alternatingly $\langle i, i-k \rangle$ and $[j, j-k]$ until $\langle i, 0 \rangle$ or $[j, 0]$
- delete $\langle i, 0 \rangle$ from the left end
- insert $\langle 0, 0 \rangle$ at the left end
- increment alternatingly $\langle k, 0 \rangle$ and $[i, k]$ until $\langle i, 0 \rangle$ and $[i, i]$
- delete $[i, i]$ from the right end
- insert $[0, 0]$ at the right end
- increment alternatingly $\langle i, k \rangle$ and $[k, k]$ until $\langle i, i \rangle$ and $[i, i]$
- replace $[i, i]$ by $l_1(i)$
- replace $\langle i, i \rangle$ by $l_2(i)$

Simulation of Rules $AB \to CD$ ($B \in N \cup \{ \lambda \}$)

Reverse derivation $B\, u_{\xi'} \# v_{\xi'} A \quad \to B u_{\xi'} \# v_{\xi'} A$

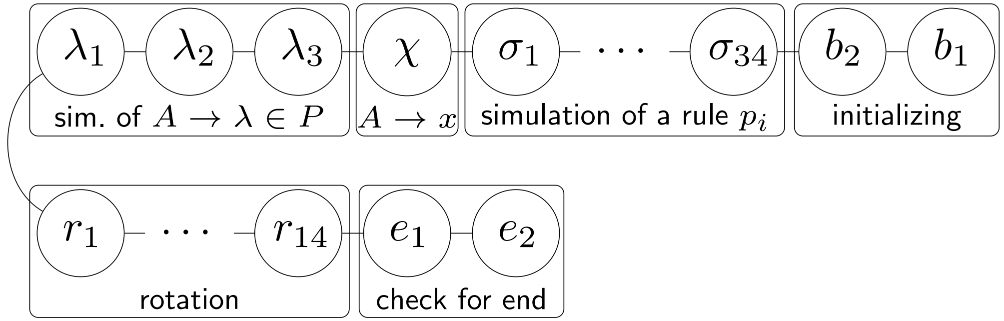
- mark $r_2(i)$ by $\langle i, i \rangle$ (hopefully at the left end)
- mark $r_1(j)$ by $[j, j]$ (hopefully at the right end)
- decrement alternatingly $\langle i, i-k \rangle$ and $[j, j-k]$ until $\langle i, 0 \rangle$ or $[j, 0]$
- delete $\langle i, 0 \rangle$ from the left end
- insert $\langle 0, 0 \rangle$ at the left end
- increment alternatingly $\langle k, 0 \rangle$ and $[i, k]$ until $\langle i, 0 \rangle$ and $[i, i]$
- delete $[i, i]$ from the right end
- insert $[0, 0]$ at the right end
- increment alternatingly $\langle i, k \rangle$ and $[k, k]$ until $\langle i, i \rangle$ and $[i, i]$
- replace $[i, i]$ by $l_1(i)$
- replace $\langle i, i \rangle$ by $l_2(i)$

Results

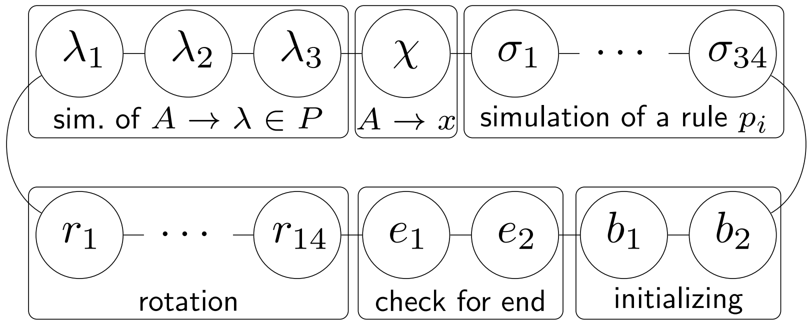


Any recursively enumerable language can be accepted by a chain with 56 nodes.

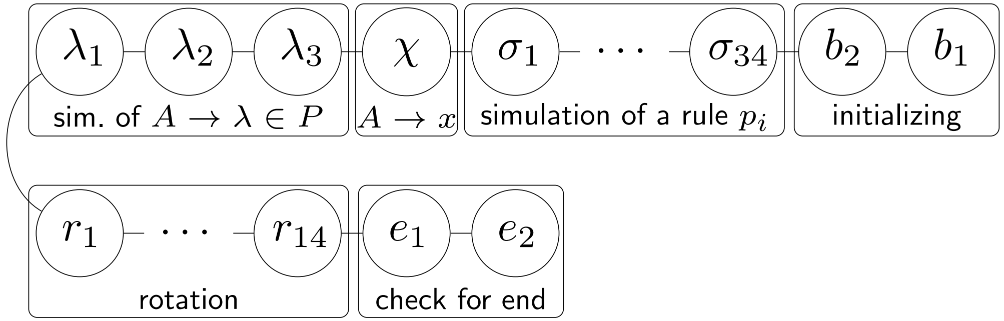
Results



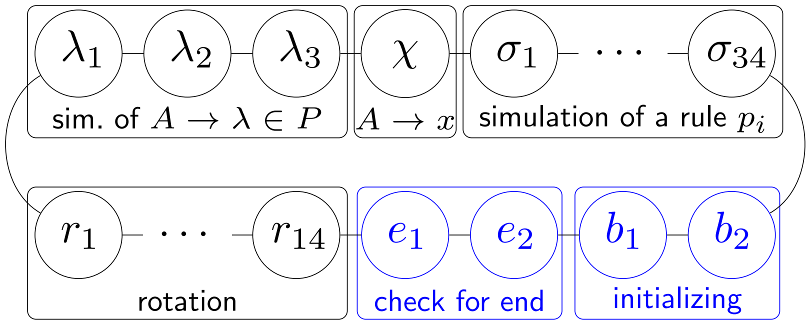
Any recursively enumerable language can be accepted by a chain with 56 nodes.

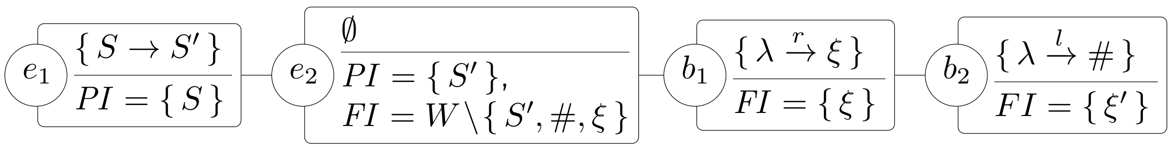


Results

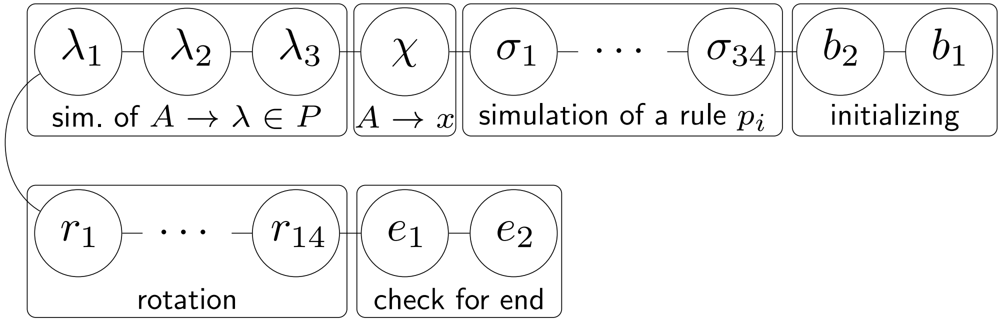


Any recursively enumerable language can be accepted by a chain with 56 nodes.

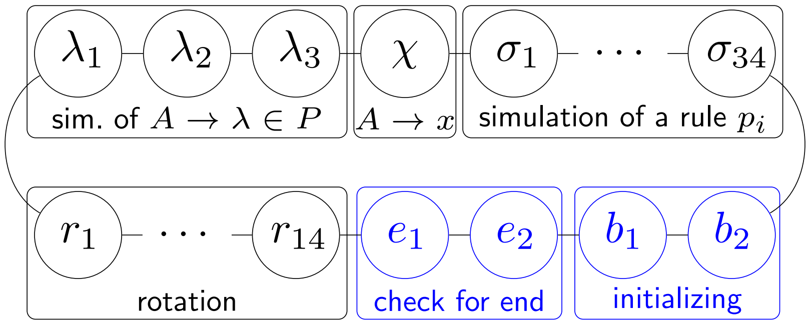


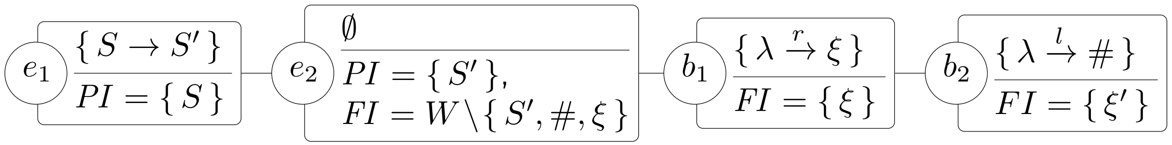


Results

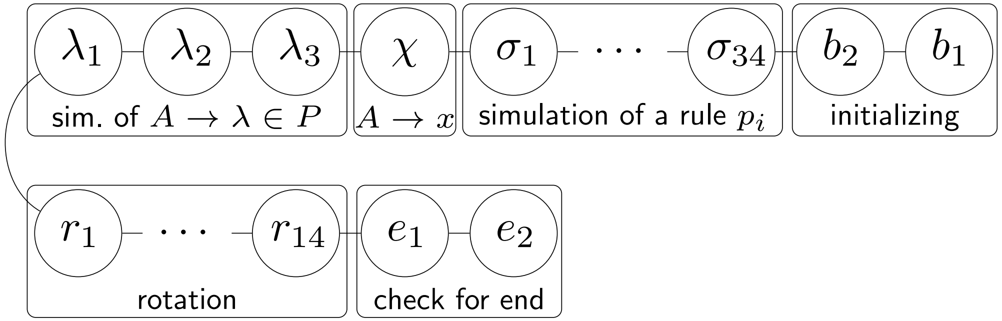


Any recursively enumerable language can be accepted by a ring with 56 nodes.

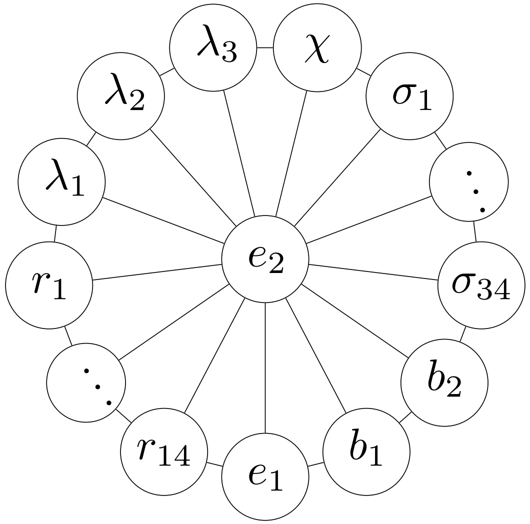




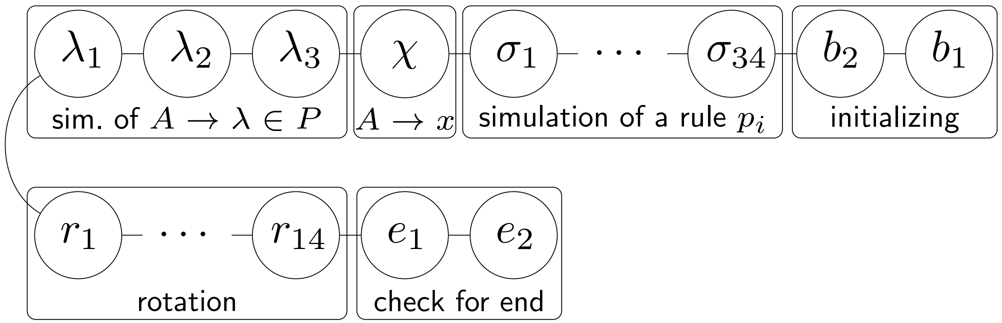
Results



Any recursively enumerable language can be accepted by a wheel with 56 nodes.



Results



Any recursively enumerable language can be accepted by a wheel with 56 nodes.