On Restarting Transducers

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Theorietag 2013, Ilmenau

Analysis by Reduction and Restarting Automata

- linguistic technique
- task: verify correctness of a given sentence

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On the extremely long run they run together

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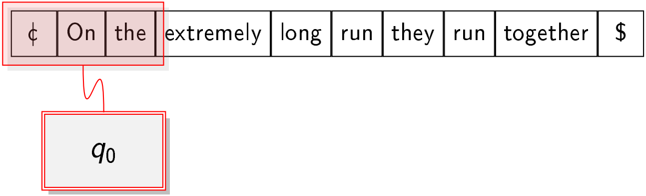

- restarting automata (Jančar, Mráz, Plátek, Vogel (1995 – 1999))

Analysis by Reduction and Relations

- analysis by reduction: correctness verification and information extracting

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On the extremely long run they run together

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$\downarrow$

depends on long

Analysis by Reduction and Relations

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On the extremely long run they run together

depends on long

noun

verb

Restarting Transducers

An RRWW-transducer
$M = (Q, \{a, b, c\}, \{a, b, c\}, \{a, b, c, B, C\}, ¢, \$, q_0, 3, \delta)$ that
computes the relation

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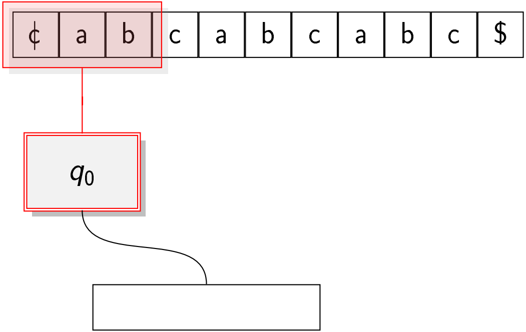

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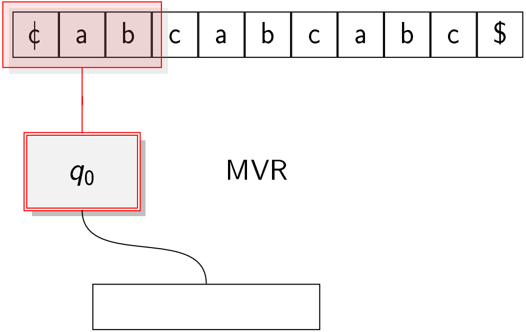

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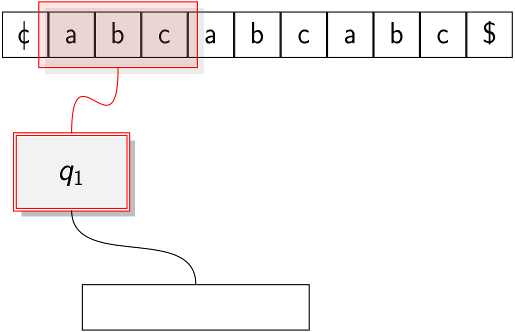

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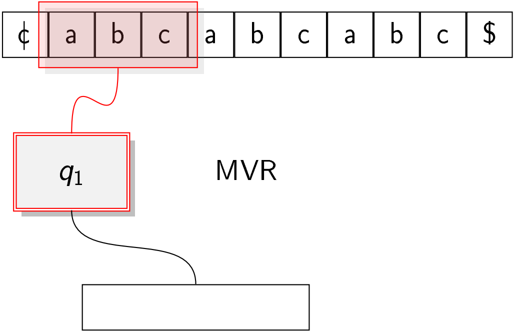

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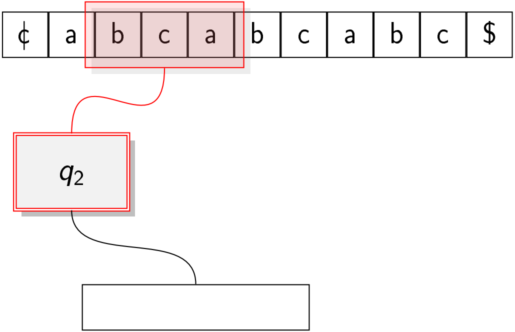

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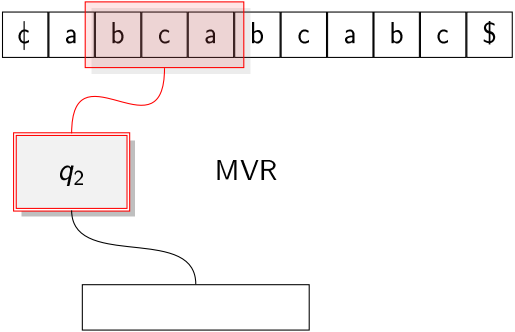

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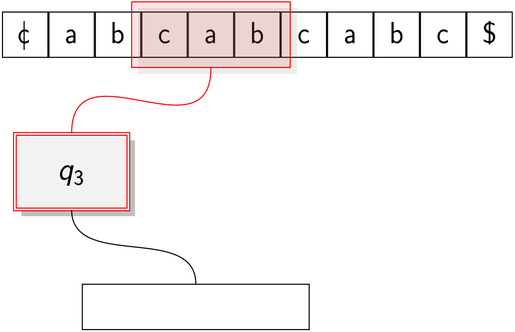

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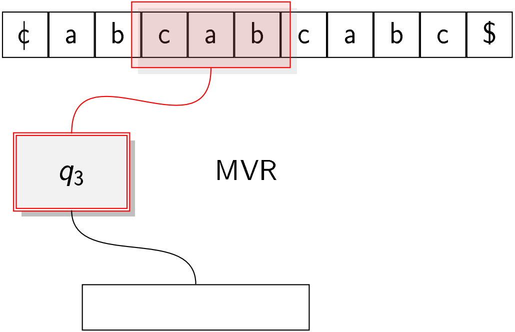

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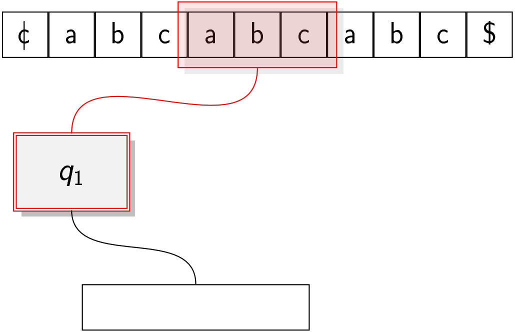

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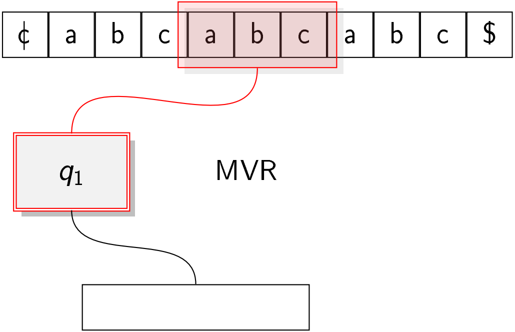

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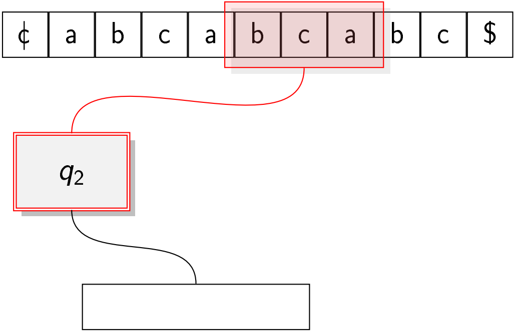

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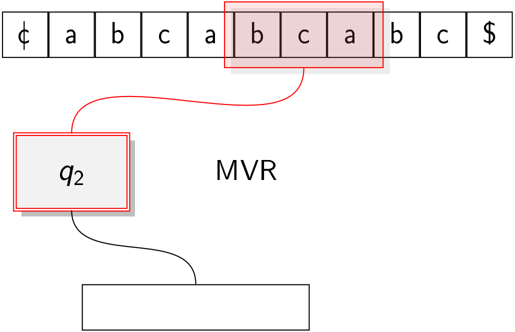

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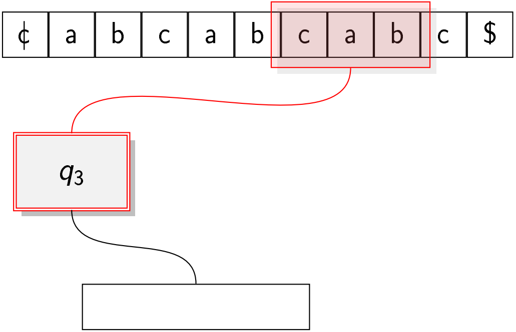

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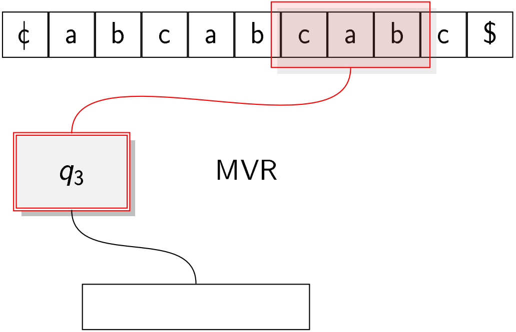

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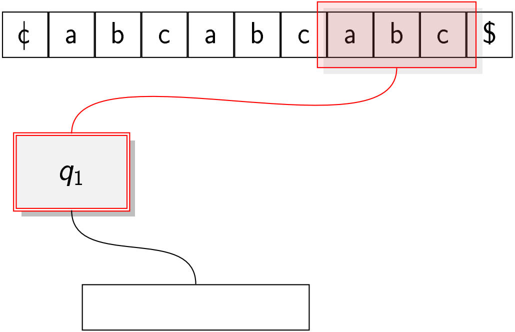

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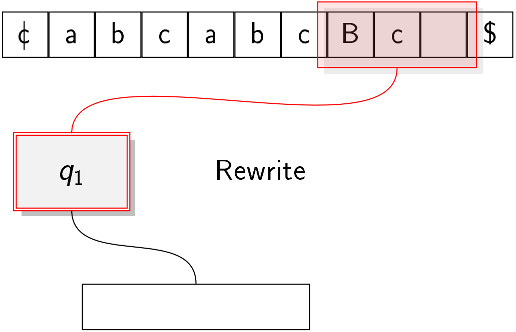

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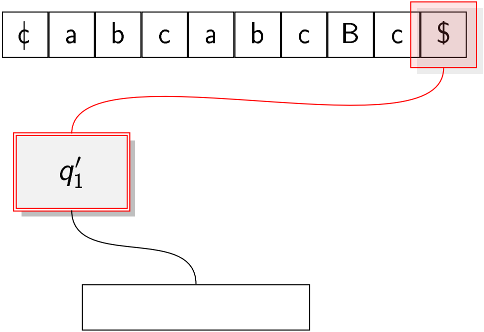

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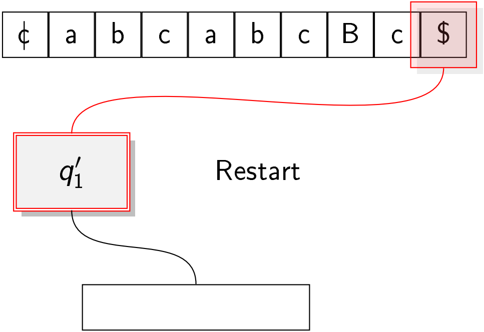

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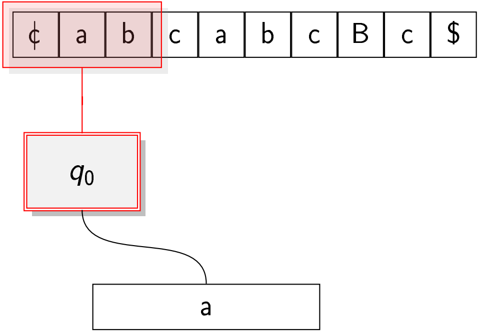

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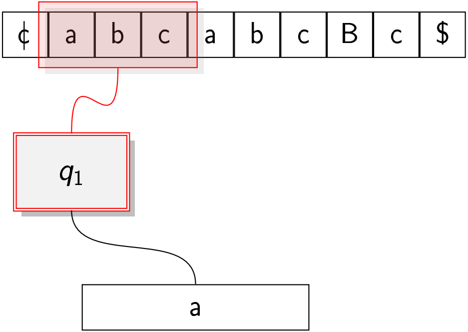

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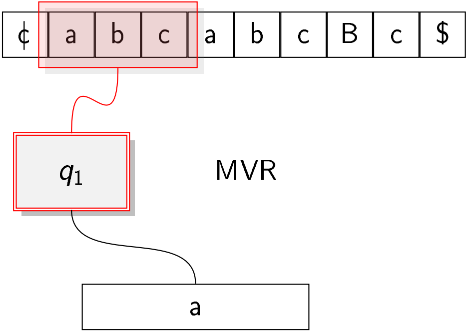

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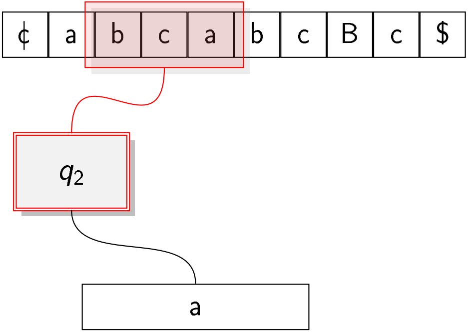

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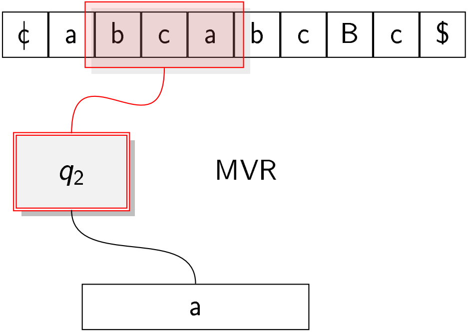

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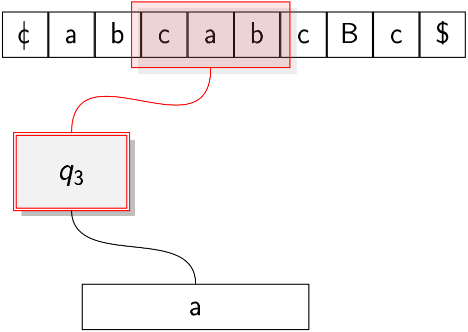

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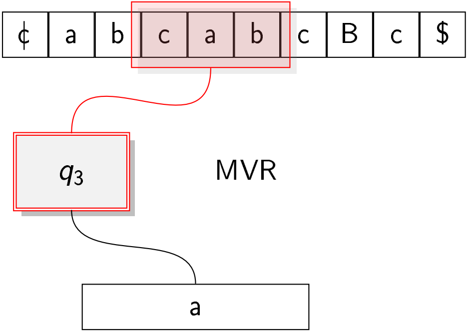

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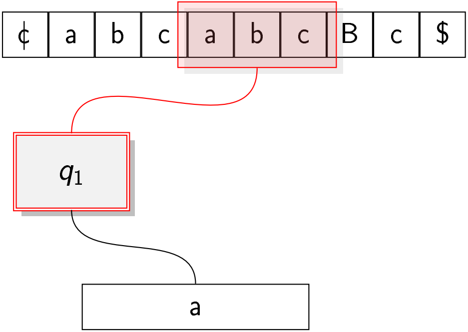

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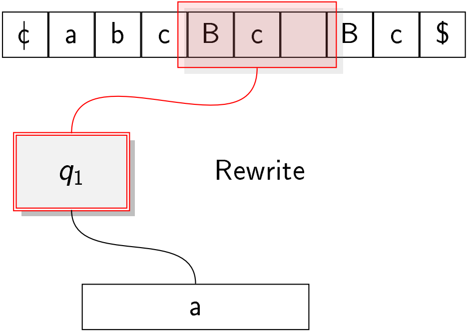

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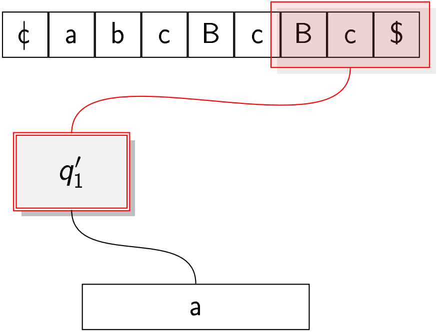

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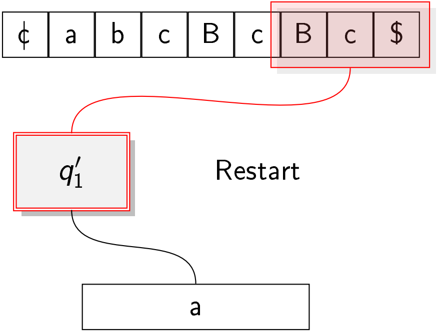

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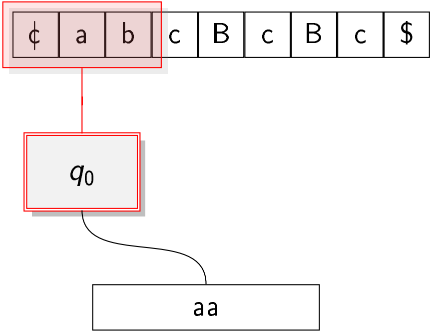

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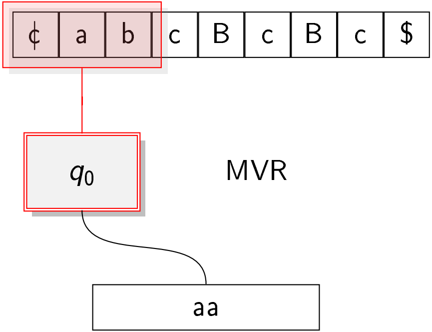

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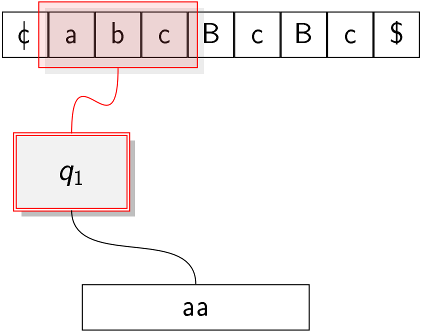

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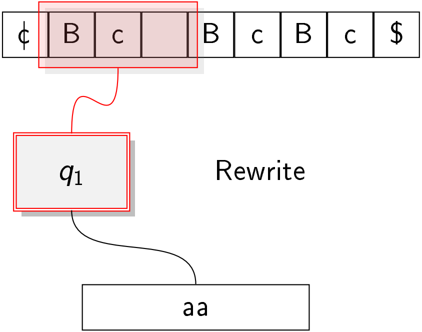

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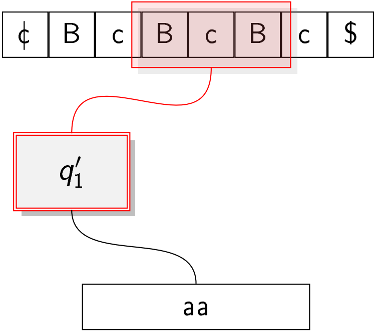

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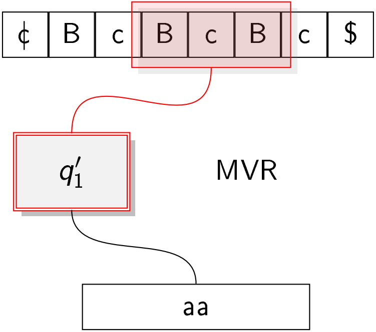

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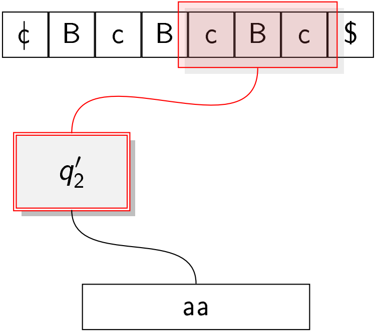

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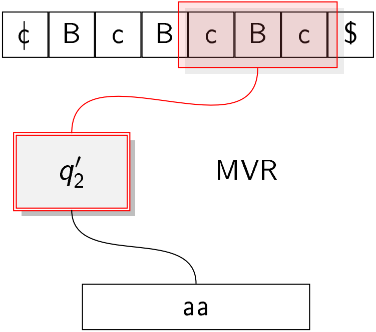

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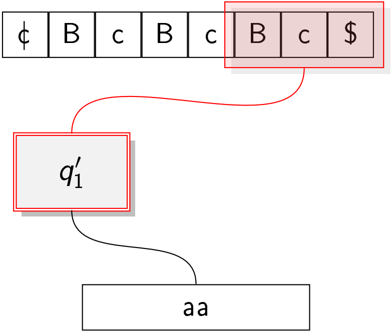
$M = (Q, \{a, b, c\}, \{a, b, c\}, \{a, b, c, B, C\}, ¢, \$, q_0, 3, \delta)$ that
computes the relation

$$\mathrm{Rel}(T) = \{((abc)^n, a^n b^n c^n) \mid n \geq 0\}.$$



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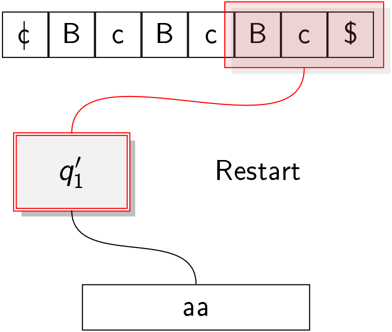

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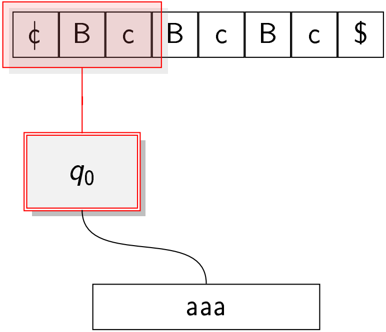

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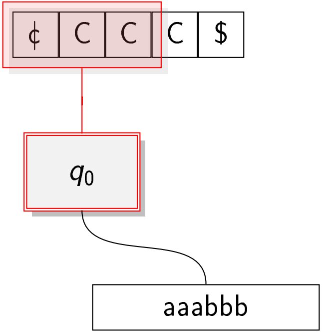

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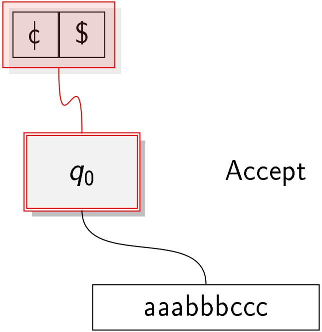
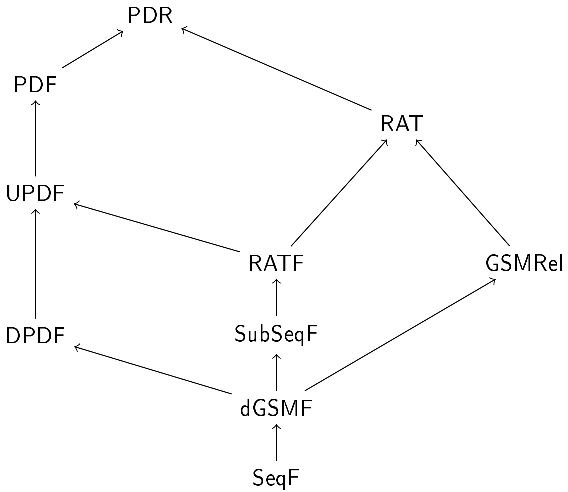


Well-Known Relation Classes



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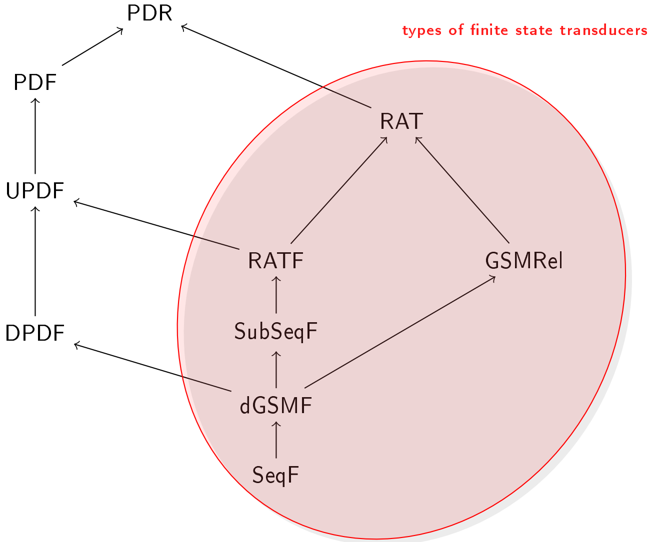

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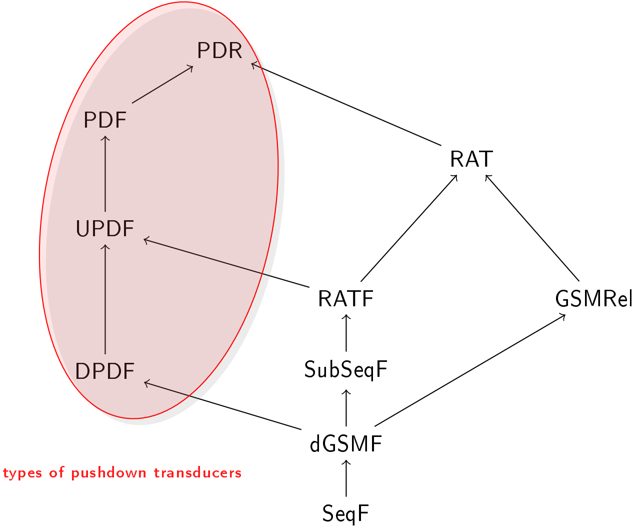

General Restarting Transducers

Observation

Every relation computed by a restarting transducer is length-bounded.

Theorem

For each recursively enumerable language L, there is a det-RWW-*transducer T such that L is the output language of T, that is, range*$(\mathrm{Rel}(T)) = L$.

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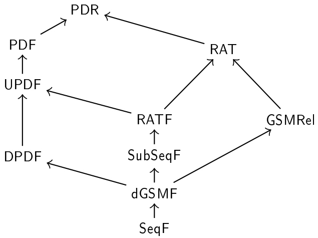
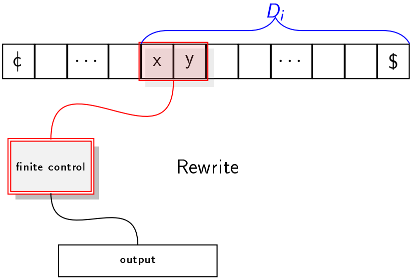


Monotone Restarting Transducers



Basis:

Theorem (Jančar, Mráz, Plátek, Vogel 1999)
$\mathcal{L}$(mon-RWW) = $\mathcal{L}$(mon-RRWW) = CFL

Question: $\mathcal{R}\mathrm{el}$(mon-RRWW-Td)?

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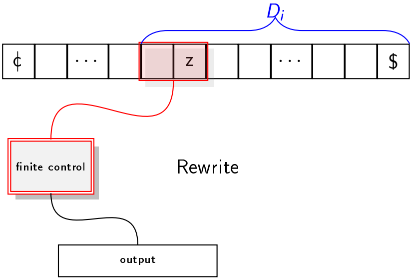

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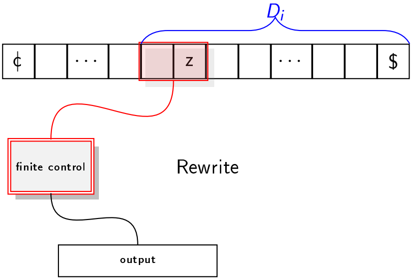

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Input-Output Relations

- $M = (Q, \Sigma', \Gamma, ¢, \$, q_0, k, \delta)$ restart automaton
- $\Sigma' = \Sigma \cup \Delta$ (Σ and Δ are disjoint)
- Pr^{Σ} and Pr^{Δ} denotes the projections from Σ'^* onto Σ^* and Δ^*

$$\boxed{\mathrm{Rel}_{io}(M) = \{(u, v) \in \Sigma^* \times \Delta^* \mid \exists w \in L(M) : u = \mathrm{Pr}^{\Sigma}(w),\ v = \mathrm{Pr}^{\Delta}(w)\}}$$

Corollary

$\mathcal{R}\mathrm{el}_{io}(\text{mon-RWW}) = \mathcal{R}\mathrm{el}_{io}(\text{mon-RRWW}) = \mathrm{PDR}$ *(pushdown relations)*

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Length-Bounded Pushdown Relations

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The class of length-bounded pushdown relations is denoted by lbPDR.

Theorem

$\text{lbPDR} = \mathcal{R}\mathrm{el}(\text{mon-RWW-Td})$

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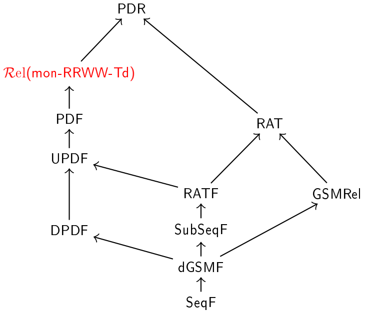
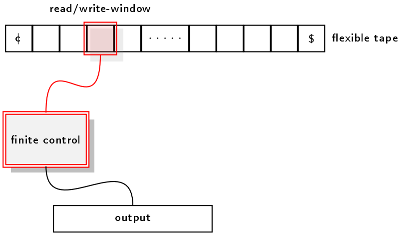


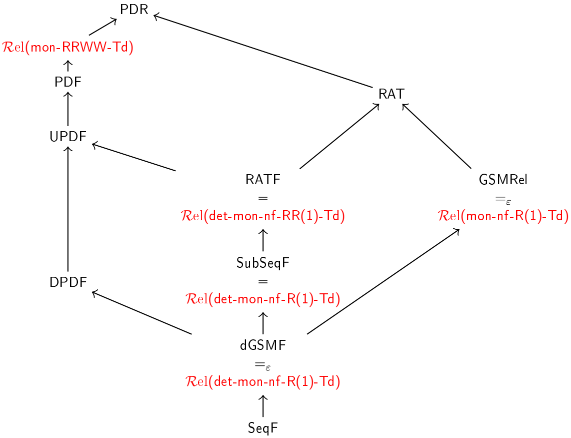
Restarting Transducer with Window Size One



- here: relations computed by monotone R(1)- and RR(1)-transducers that accept only regular languages

=> form a hierarchy in the rational relations

Summary


PDR
$\mathcal{R}\mathrm{el}$(mon-RRWW-Td)
PDF
UPDF
RAT
RATF
=
$\mathcal{R}\mathrm{el}$(det-mon-nf-RR(1)-Td)
GSMRel
$=_\varepsilon$
$\mathcal{R}\mathrm{el}$(mon-nf-R(1)-Td)
SubSeqF
=
$\mathcal{R}\mathrm{el}$(det-mon-nf-R(1)-Td)
DPDF
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SeqF

Thank you for your attention!