Input-Driven Queue Automata: Finite Turns, Decidability, and Closure Properties

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- Extensions/generalizations: multiple pushdowns, graph automata, height-deterministic PDA, stacks, . . .

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- Deterministic (DQA) and nondeterministic variants.
- Extended variants with several queues.
- Undecidability of emptiness for deterministic queue automata working in real time.

Input-driven queue automata (DVQA)

$M = \langle Q, \Sigma, \Gamma, q_0, F, \perp, \delta_e, \delta_r, \delta_i \rangle$,

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By definition, DVQA work in real time.

Example: The language

$$\{\,\$_0\$_1 abb\$_2\$_1 abbabb\$_2\$_1 (abb)^4\$_2 \ldots \$_1 (abb)^{(2^n)}\$_2 \mid n \geq 0\,\}$$

is accepted by the following DVQA.

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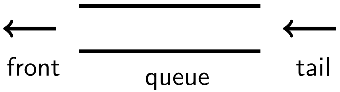

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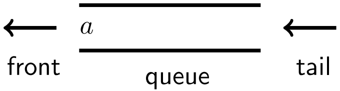

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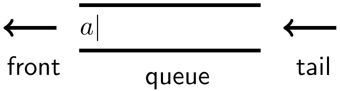

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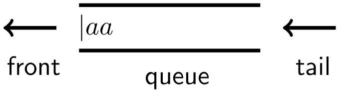

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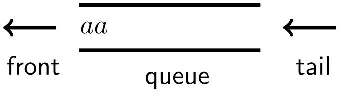

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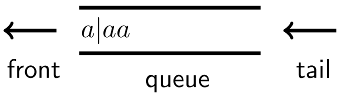

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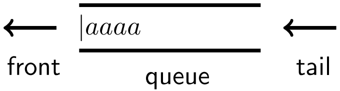

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- For any given $k \geq 0$, a k-turn computation is any computation containing exactly k turns.
- We restrict deterministic queue automata, to make at most k turns in the queue (DQA_k and DVQA_k).

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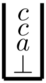

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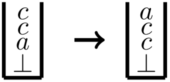

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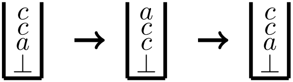

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- The idea of the construction is to use one end of the pushdown store as the front and the other end as the tail of the queue.
- Whenever the queue automaton performs a turn, that is, changes from increasing to decreasing or decreasing to increasing mode, the flip-pushdown automaton flips the front end of the pushdown store to the top.

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- So every language accepted by a queue automaton with a constant number of turns obeys a semilinear Parikh mapping.

Theorem

Let $k \geq 0$ be a constant and M be a k-turn DQA. Then $L(M)$ is semilinear, in particular, if $L(M)$ is a unary language then it is regular.

Turn hierarchy

Example: Let $h_p : \{a, b\}^* \to \{a', b'\}^*$ be the homomorphism $h_p(a) = a'$, $h_p(b) = b'$. For all $j \geq 0$, we define the sets

$$C_j = \{\, \#w\#h_p(w) \mid w \in \{a, b\}^* \,\}^j \cdot \#$$

and, for all $k \geq 0$ the language $L_k = \bigcup_{j=0}^{k} C_j$.

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Theorem

Let $k \geq 1$. Then language L_k is accepted by some DVQA_k, but not accepted by any DQA_{k-1}.

Closure properties

	DVQA$_k$	DVQA
$\sim$	no	yes
$\cup_c$	yes	yes
$\cap_c$	yes	yes
$\cap_{REG}$	yes	yes
$\cdot$	no	no
$*$	no	no
h_λ	no	no
h^{-1}	no	no
$\cup$	no	no
$\cap$	no	no

Two signatures
$\Sigma = \Sigma_e \cup \Sigma_r \cup \Sigma_i$
and
$\Sigma' = \Sigma'_e \cup \Sigma'_r \cup \Sigma'_i$
are compatible if
$\bigcup_{j \in \{e,r,i\}} (\Sigma_j \setminus \Sigma'_j) \cap \Sigma' = \emptyset$
and
$\bigcup_{j \in \{e,r,i\}} (\Sigma'_j \setminus \Sigma_j) \cap \Sigma = \emptyset$.

Decidability problems

	DVQA_k	DVQA
emptiness		
finiteness		
universality		
inclusion		
inclusion$_c$		
equivalence		
equivalence$_c$		
finite turn		

- $+$ means *decidable*
- $-$ means *not semidecidable*

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universality	+	
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inclusion_c	+	
equivalence		
equivalence_c	+	
finite turn		

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Decidability problems

Lemma

Let M be an LBA. Then a DVQA accepting $\mathrm{VALC}(M)$ can effectively be constructed.

Decidability problems

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emptiness	$+$	$-$
finiteness	$+$	$-$
universality	$+$	$-$
inclusion		$-$
inclusion$_c$	$+$	$-$
equivalence		$-$
equivalence$_c$	$+$	$-$
finite turn		

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finiteness	$+$	$-$
universality	$+$	$-$
inclusion	$-$	$-$
inclusion$_c$	$+$	$-$
equivalence		$-$
equivalence$_c$	$+$	$-$
finite turn		

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finiteness	$+$	$-$
universality	$+$	$-$
inclusion	$-$	$-$
inclusion$_c$	$+$	$-$
equivalence		$-$
equivalence$_c$	$+$	$-$
finite turn	trivial	$-$

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	DVQA_k	DVQA
emptiness	$+$	$-$
finiteness	$+$	$-$
universality	$+$	$-$
inclusion	$-$	$-$
inclusion$_c$	$+$	$-$
equivalence	?	$-$
equivalence$_c$	$+$	$-$
finite turn	trivial	$-$

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- $-$ means *not semidecidable*