A Bottom-Up Deterministic Weighted Tree Automaton for the *n*-Gram Yield Function

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Outline

1. Motivation: Statistical Machine Translation
2. Generalized n-Gram Models
3. Lifting using Bar-Hillel, Perles, Shamir Algorithm
4. Generalized n-Gram Weighted Tree Automaton
5. Further Research

Motivation: Statistical Machine Translation

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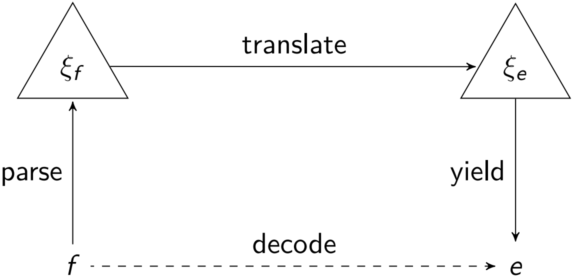
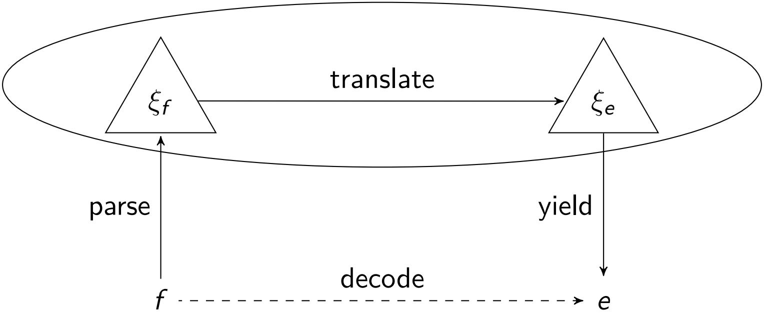


Motivation: Statistical Machine Translation

TM: $T_\Delta \times T_\Sigma \to \mathbb{R}_{\geq 0}$



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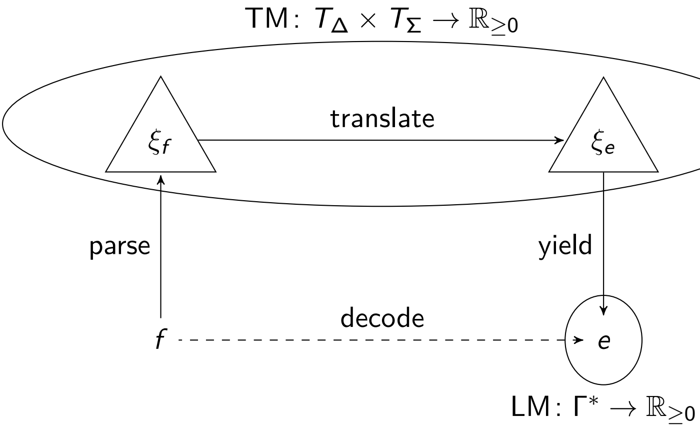

$\mathbf{\Gamma} = \mathbf{\Sigma}^{(0)}$

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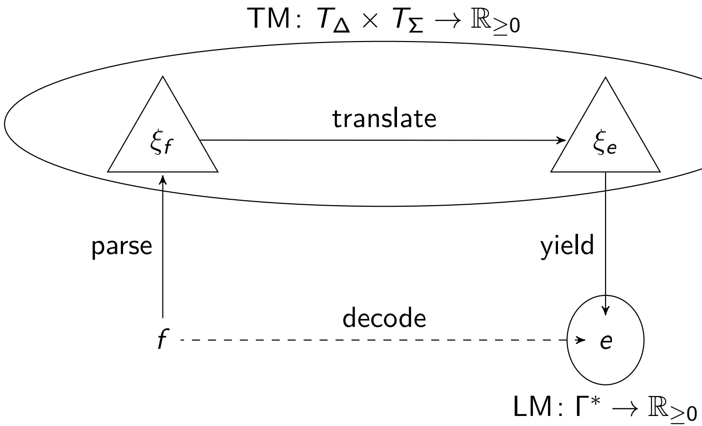

$$\hat{e} = \pi_{\text{out}}\Big(\text{argmax}_d \,\big(\,(f \triangleleft \text{TM}) \,\triangleright\, \text{LM}\,\big)(d)\Big)$$

$\mathbf{\Gamma} = \mathbf{\Sigma}^{(0)}$

Generalized n-Gram Models

Let $n \in \mathbb{N} \setminus \{0\}$.

A *generalized n-gram model* is a tuple $N = (\Gamma, \mu_1, \ldots, \mu_n)$ with

- the alphabet Γ
- a mapping $\mu_i \colon \Gamma^i \to \mathbb{R}_{\geq 0}$ for every $i \in \{1, \ldots, n\}$

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The semantics of N is $[\![N]\!] \colon \Gamma^* \to \mathbb{R}_{\geq 0} \cup \{\bot\}$ where

$$[\![N]\!](w) = \begin{cases} \prod_{w' \text{ } n\text{-gram of w}} \mu_n(w') & \text{if } |w| \geq n \\ \mu_{|w|}(w) & \text{if } 1 \leq |w| < n \\ \bot & \text{if } |w| = 0 \end{cases}$$

Generalized n-Gram Models (Example)

Let $n = 3$, $\Gamma = \{\text{Garcia}, \text{y}, \text{tres}, \text{asociados}, \textbf{.}\}$, and

$$\begin{aligned} \mu_3\colon\ & \text{Garcia y tres} \mapsto 1/5 & \mu_2\colon\ & \text{Garcia } \textbf{.} \mapsto 1/2 & \ldots \\ & \text{y tres asociados} \mapsto 1/3 & & \ldots \\ & \text{tres asociados } \textbf{.} \mapsto 1/4 \\ & \ldots \end{aligned}$$

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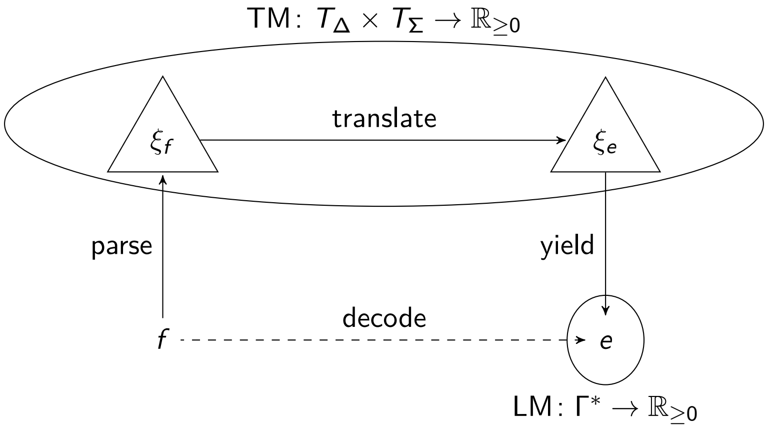
$$[\![N]\!](\varepsilon) = \bot$$

Recall: Statistical Machine Translation



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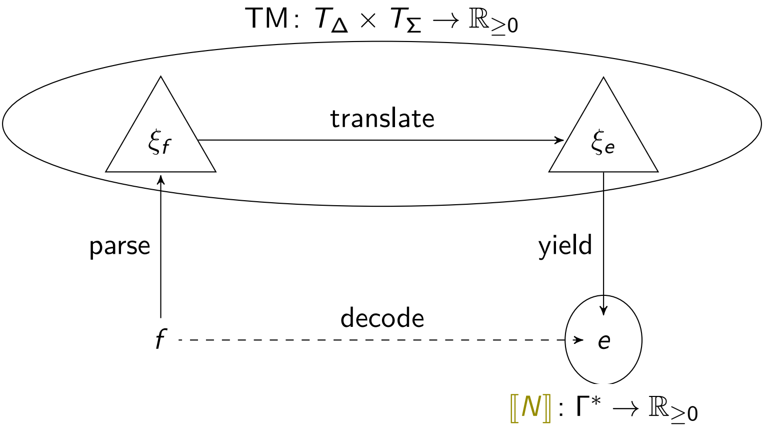

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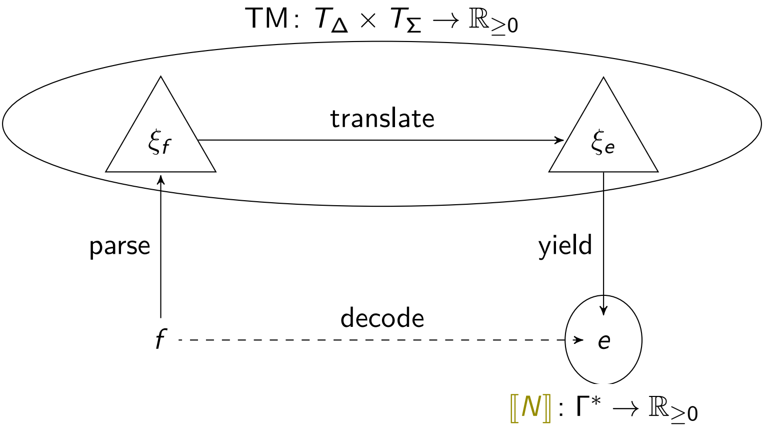

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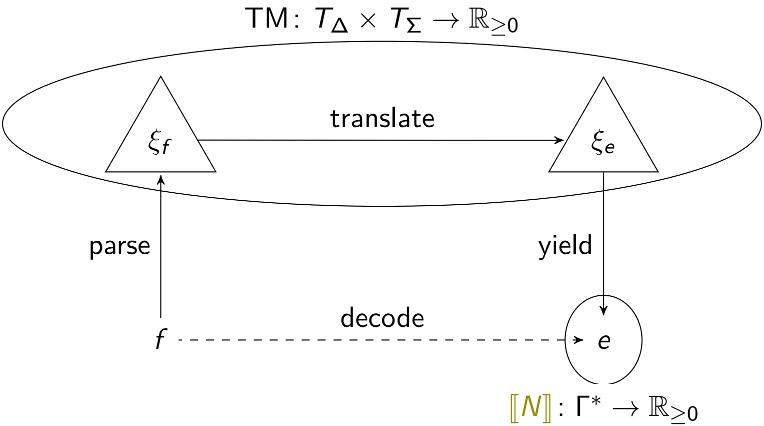

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$T_\Delta \times T_\Sigma \to \mathbb{R}_{\geq 0}$ $\Gamma^* \to \mathbb{R}_{\geq 0}$

lift to trees
$\text{yield}^{-1}([\![N]\!])$

Lifting using Bar-Hillel, Perles, Shamir (BHPS)

- for every $\mathcal{L} \in \mathrm{CF}$ and $R \in \mathrm{REG}$: $\mathcal{L} \cap R \in \mathrm{CF}$

[Bar-Hillel, Perles, Shamir, 1961]

CF . . . context-free languages
REG . . . regular languages

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  [Bar-Hillel, Perles, Shamir, 1961]

- for every $\mathcal{L} \in \mathrm{WRT}$ and $R \in \mathrm{WREG}$: $\mathcal{L} \odot \mathrm{yield}^{-1}(R) \in \mathrm{WRT}$

  [Maletti, Satta, 2009]

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WRT . . . weighted regular tree languages
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  $\mathrm{yield}^{-1}([\![N]\!]) \in \mathrm{WRT}$

  - the constructed WTA is *not* bottom-up deterministic
  - *but* for every tree, only one run is relevant

CF . . . context-free languages
REG . . . regular languages
WTA. . . weighted tree automaton
WRT . . . weighted regular tree languages
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Intermezzo: Weighted Tree Automata (WTA)

A *weighted tree automaton* is a tuple $\mathcal{A} = (Q, \Sigma, \delta, F)$ with

- Q finite set (states)
- Σ ranked alphabet (input symbols)
- $\delta = (\delta_k \mid k \geq 0)$ with $\delta_k \colon Q^k \times \Sigma^{(k)} \times Q \to \mathbb{R}_{\geq 0}$ (transition function)
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The *weight of κ* is $\text{wt}(\kappa) = \prod_{\rho \in \text{pos}(\kappa)} \delta_k(\kappa(\rho 1), \ldots, \kappa(\rho k), \sigma, \kappa(\rho))$.

$\boldsymbol{\sigma}$ . . . label in ξ at ρ $\quad$ $\boldsymbol{k}$ . . . rank of σ

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The *semantics of $\mathcal{A}$* is $[\![\mathcal{A}]\!] \colon T_\Sigma \to \mathbb{R}_{\geq 0}$ where

$$[\![\mathcal{A}]\!](\xi) = \sum_{\substack{\kappa \text{ run on } \xi \\ \kappa(\varepsilon) \in F}} \text{wt}(\kappa) \ .$$

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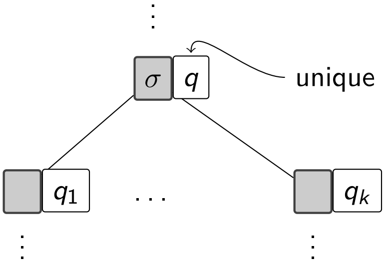
Intermezzo: Weighted Tree Automata (WTA)

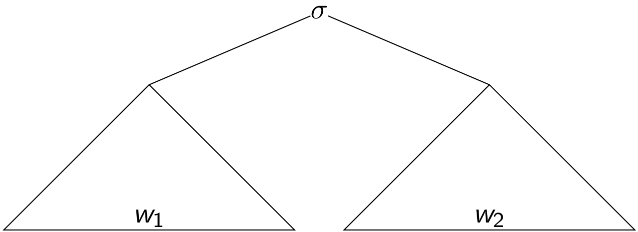
$\mathcal{A}$ is *bottom-up deterministic*:



$$\delta_k(q_1, \ldots, q_k, \sigma, q) > 0$$

Generalized n-Gram WTA (Idea)

$n = 3$



$[\![N]\!](w_1 w_2) =$ · ·

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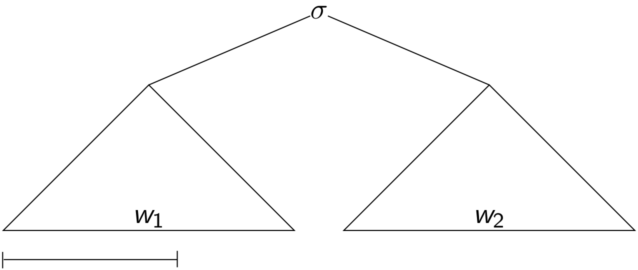

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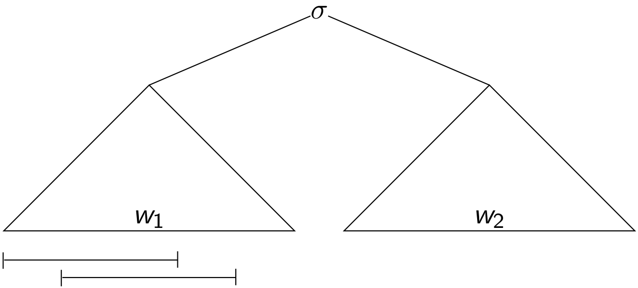

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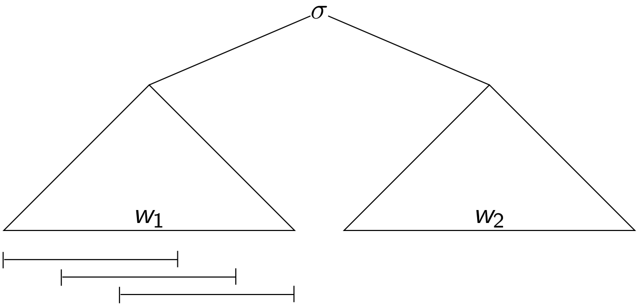

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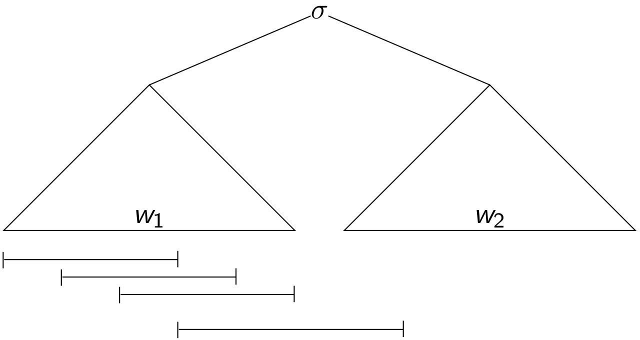

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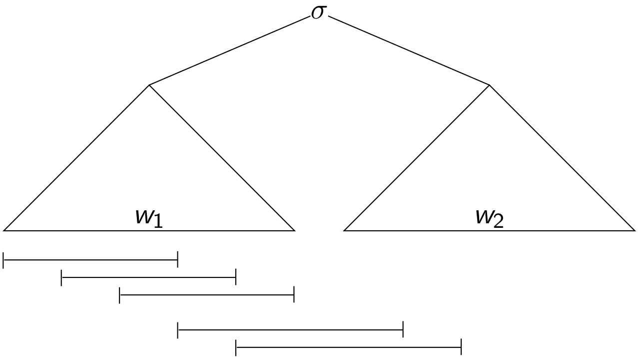

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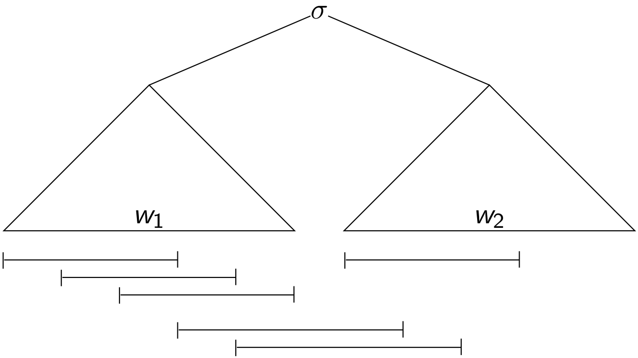

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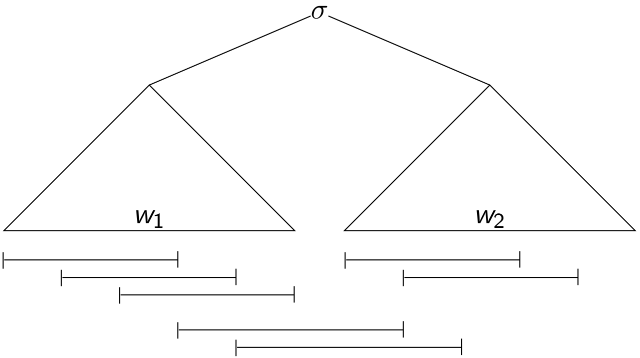

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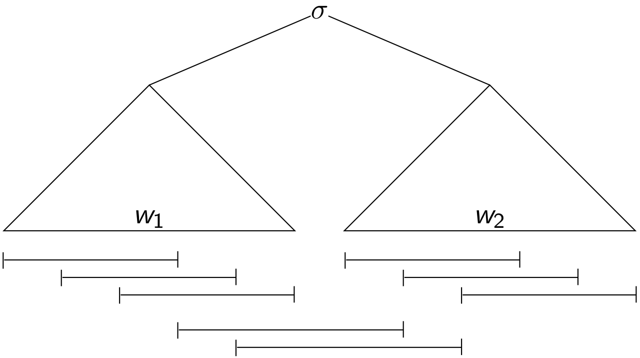

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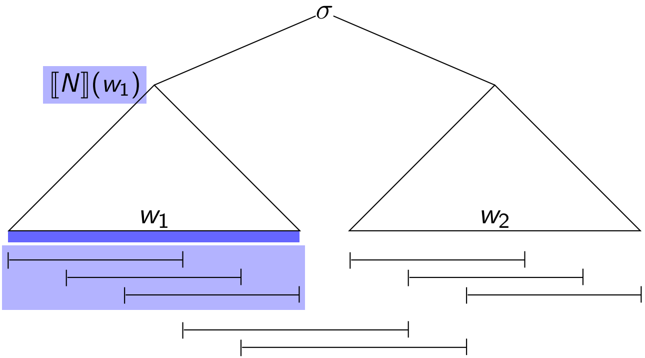

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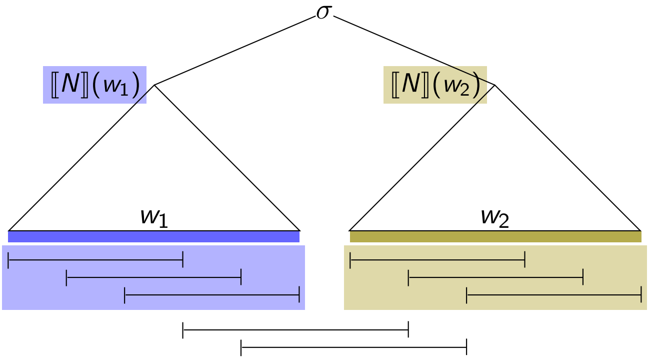

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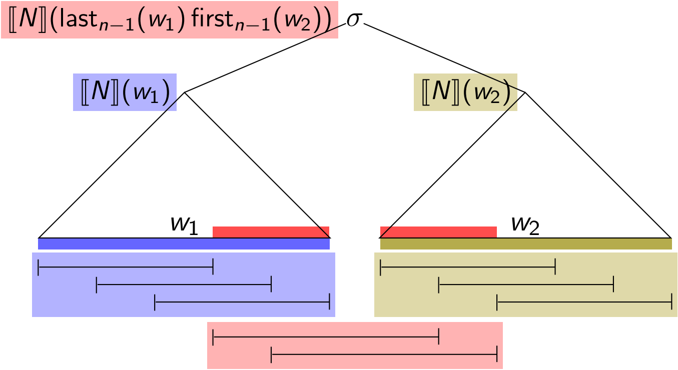

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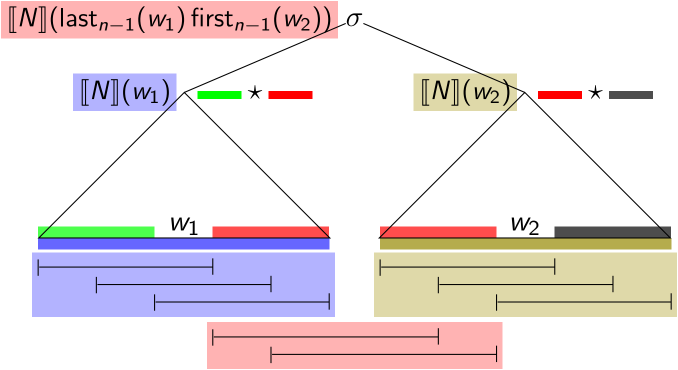

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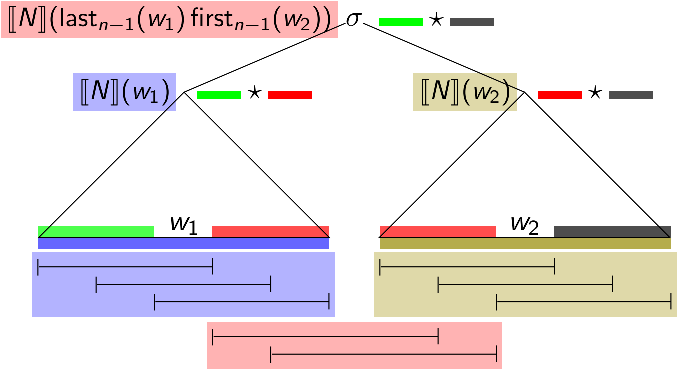

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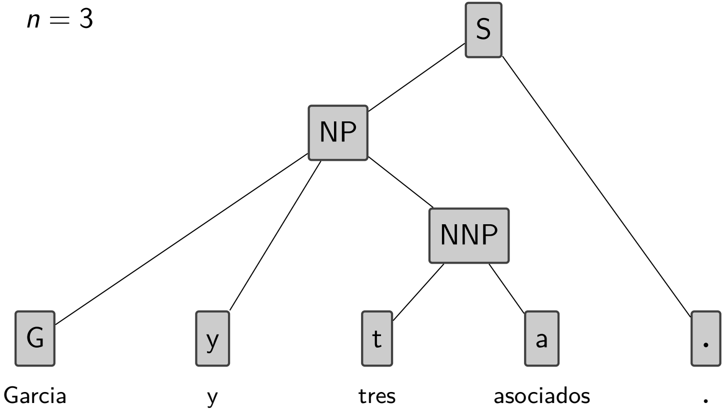

$\text{wt}(\kappa) =$

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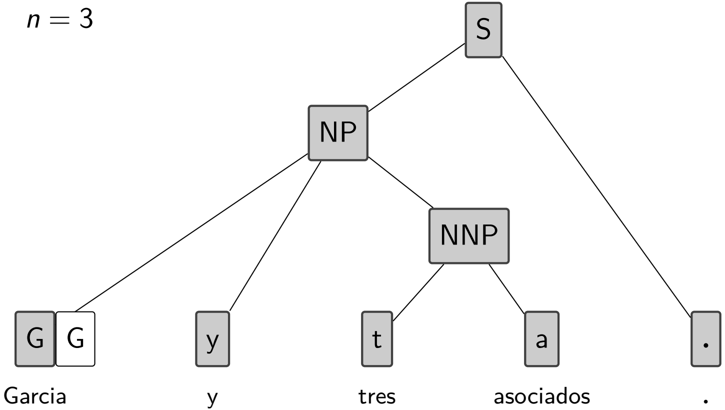

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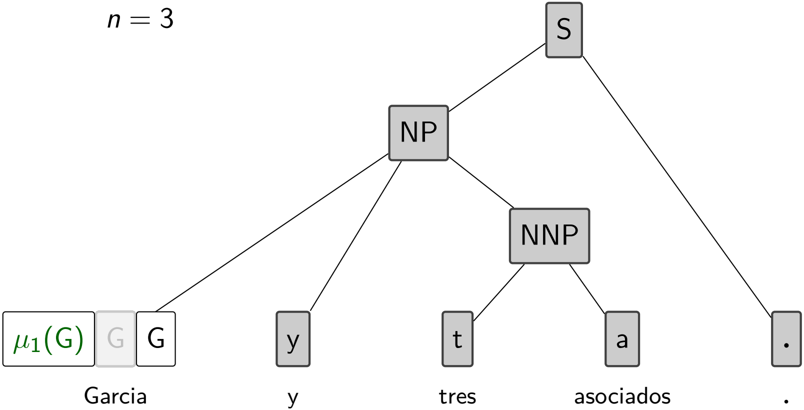

$\text{wt}(\kappa) = \mu_1(\text{G})$

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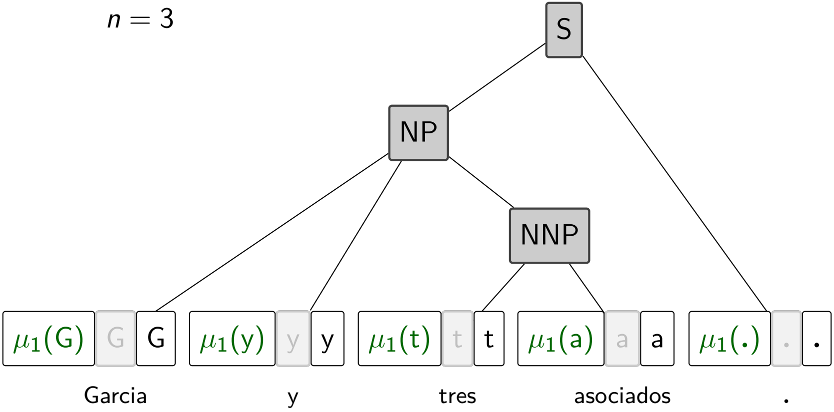

$\mathrm{wt}(\kappa) = \mu_1(\mathrm{G}) \cdot \mu_1(\mathrm{y}) \cdot \mu_1(\mathrm{t}) \cdot \mu_1(\mathrm{a}) \cdot \mu_1(.)$

Generalized n-Gram WTA (Example)

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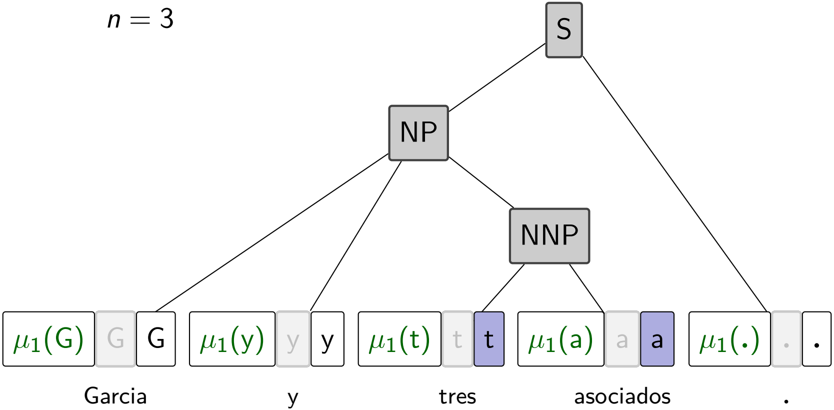

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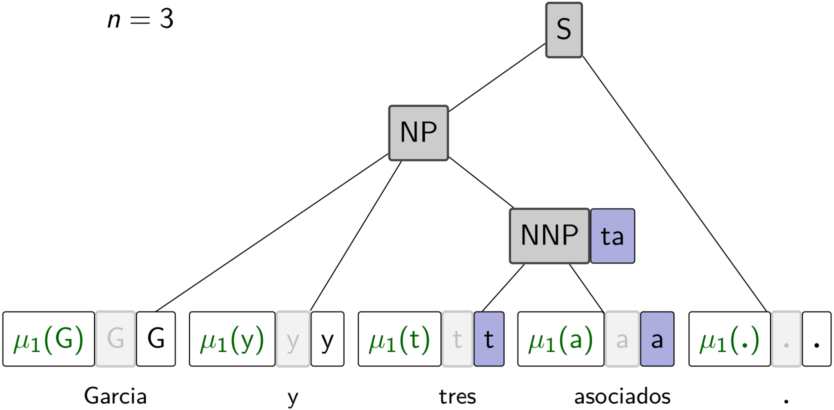

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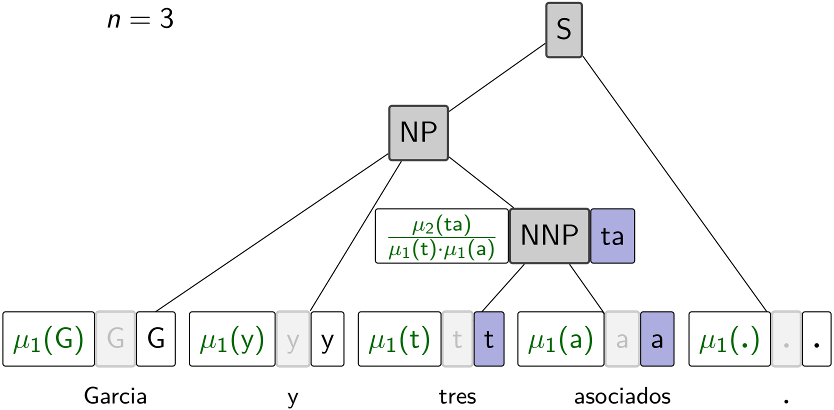

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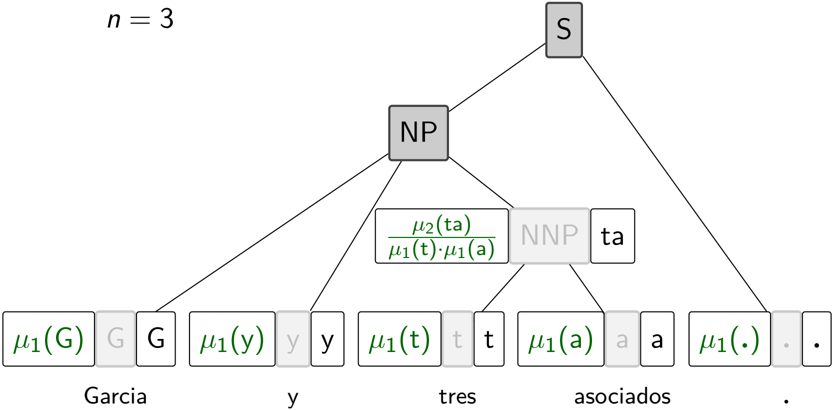

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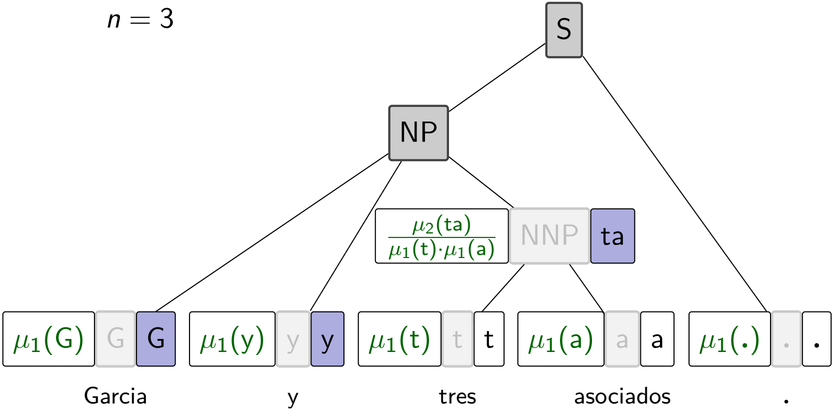

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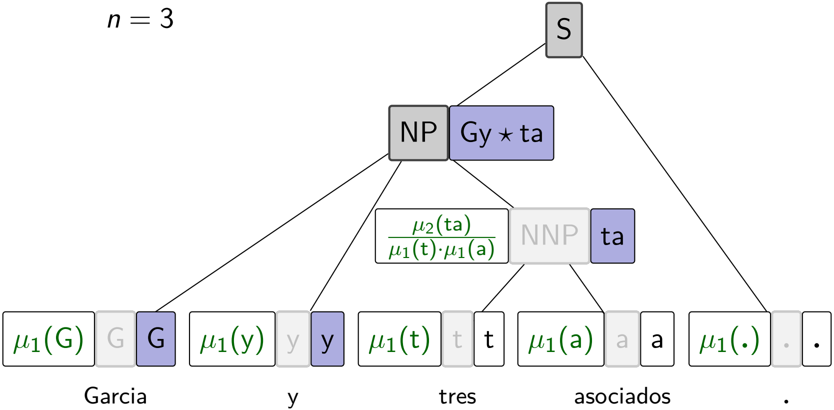

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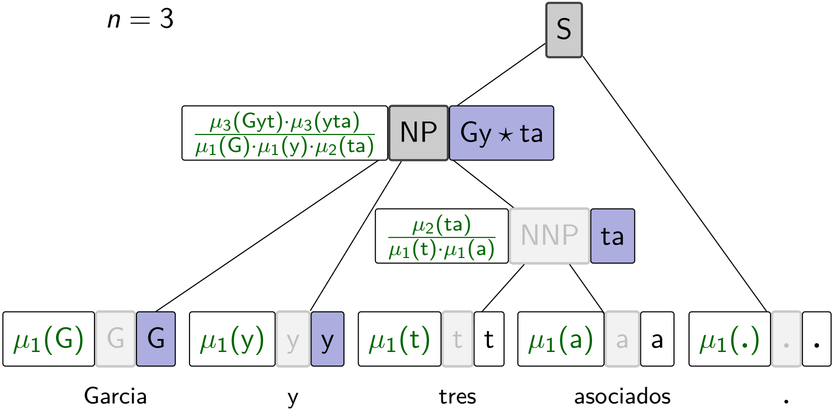

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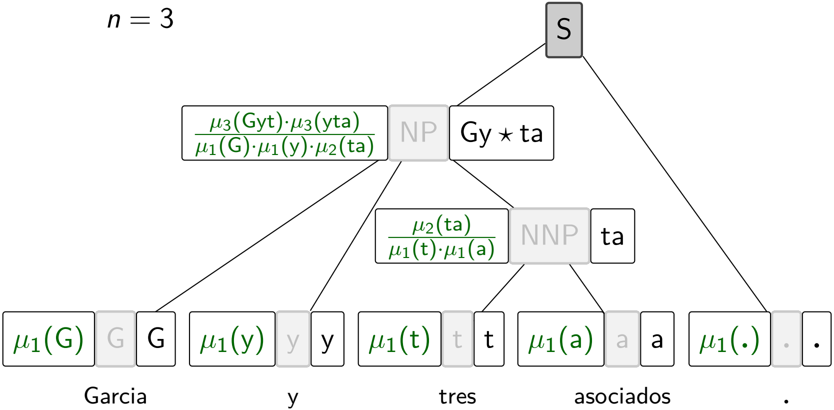

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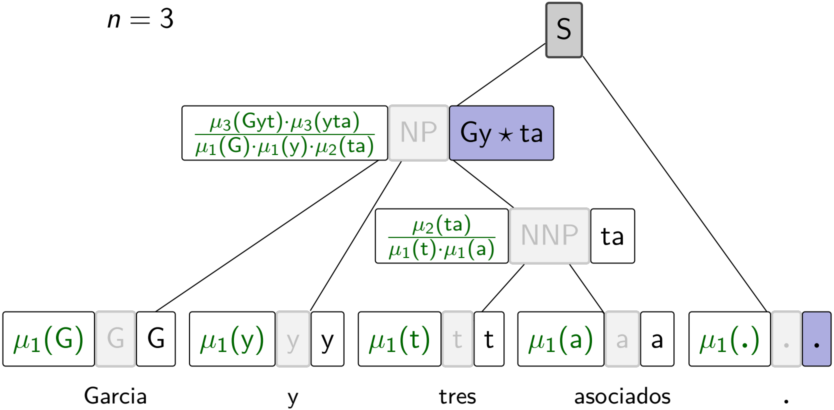

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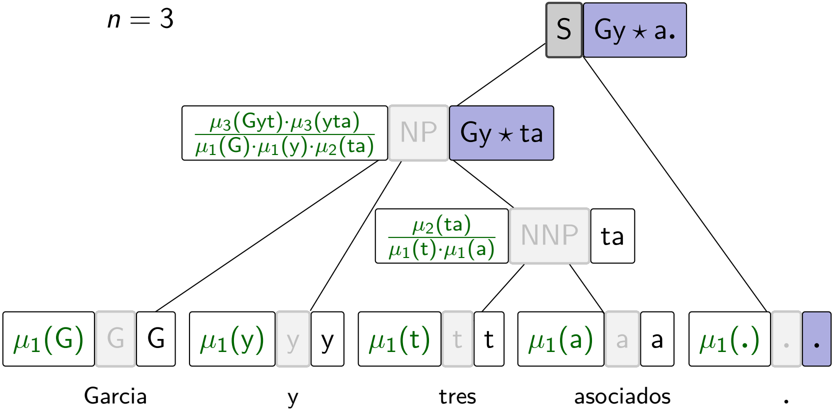

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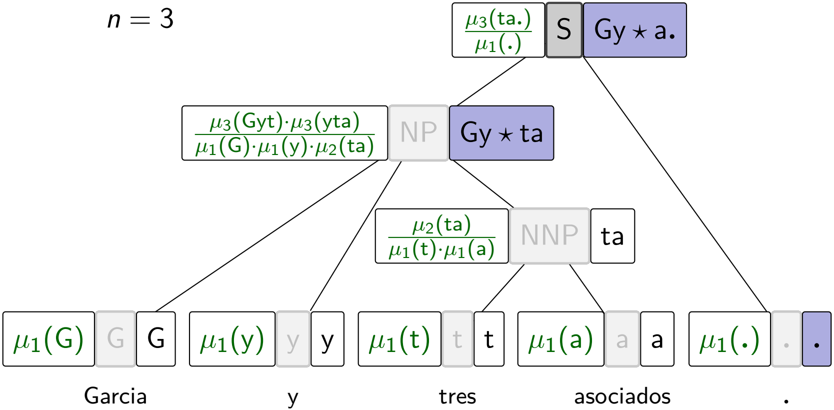

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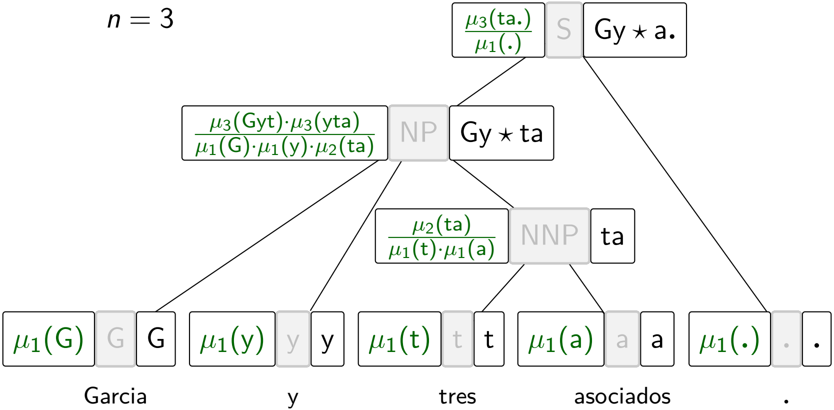

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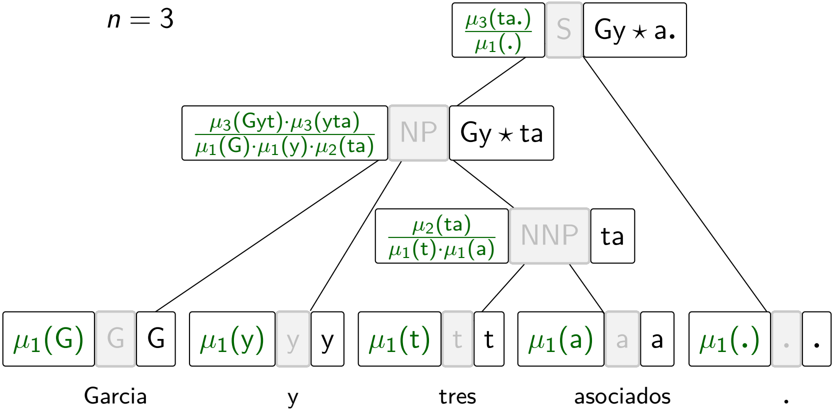

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The WTA $\mathcal{A}_{N,\Sigma}$ is the tuple (Q, Σ, δ, F) with

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$$f(v) = \begin{cases} v & \text{if } v \in \Gamma^* \text{ and } |v| < n \\ v_1^{n-1} \star v_{|v|-n+2}^{|v|} & \text{if } (v \in \Gamma^* \text{ and } |v| \geq n) \text{ or} \\ & \quad (v \notin \Gamma^* \text{ and } |v| \geq n) \\ \ldots & \text{otherwise} \end{cases}$$

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  for every $v \in (\Gamma \cup \{\star\})^*$, $v = u_0 \star u_1 \cdots \star u_l$, $u_i \in \Gamma^*$ for all $i \in \{0, \ldots, l\}$, $w \in \Sigma^*$, and $q \in Q$

$$g(v) = \begin{cases} [\![N]\!](u_0) & \text{if } l = 0 \\ N'(u_0) \cdot \left(\prod_{i=1}^{l-1} [\![N]\!](u_i)\right) \cdot N'(u_l) & \text{otherwise} \end{cases}$$

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Theorem

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Let Σ be a ranked alphabet and N a generalized n-gram model over Γ. Then

- $[\![\mathcal{A}_{N,\Sigma}]\!] = \text{yield}^{-1}([\![N]\!])$
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Conjecture

$\mathcal{A}_{N,\Sigma}$ is well suited for the calculation of $\mathrm{argmax}_d\,((f \triangleleft \mathrm{TM}) \triangleright [\![N]\!])(d)$.

Further Research

- Implement a decoder for SMT based on
  - BHPS, and
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- Compare the runtime behaviour in practice.

References

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BHSP for Trees and n-Gram Models

$\mathcal{L} = T_\Sigma$ **as WTA**

$Q_1 = \{\star\}$

for every $k \in \mathbb{N}, \sigma \in \Sigma^{(k)}$:

$$\star \xrightarrow{1} \sigma(\underbrace{\star, \ldots, \star}_{k \textbf{ times}})$$

WFSA... weighted finite state automaton $\quad \mathbf{\Gamma} = \mathbf{\Sigma}^{(0)}$

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$Q_2 = \Gamma^0 \cup \cdots \cup \Gamma^{n-1}$

for every $q \in Q \setminus \Gamma^{n-1}, \sigma \in \Gamma$:

$$q \xrightarrow{\sigma/1} q\sigma$$

for every $q \in \Gamma^{n-2}, \sigma, \sigma' \in \Gamma$:

$$\sigma' q \xrightarrow{\sigma/\mu(\sigma' q \sigma)} q\sigma$$

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$\mathcal{L} \odot R$ **as WTA**

$Q = Q_2 \times Q_1 \times Q_2$

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$$q \star q\sigma \xrightarrow{1} \sigma, \qquad \sigma' q \star q\sigma \xrightarrow{\mu(\sigma' q \sigma)} \sigma$$

for every $k \in \mathbb{N}, \sigma \in \Sigma^{(k)} \setminus \Gamma, q_0, \ldots, q_k \in Q_2$:

$$q_0 \star q_k \xrightarrow{1} \sigma(q_0 \star q_1, q_1 \star q_2, \ldots, q_{k-1} \star q_k)$$

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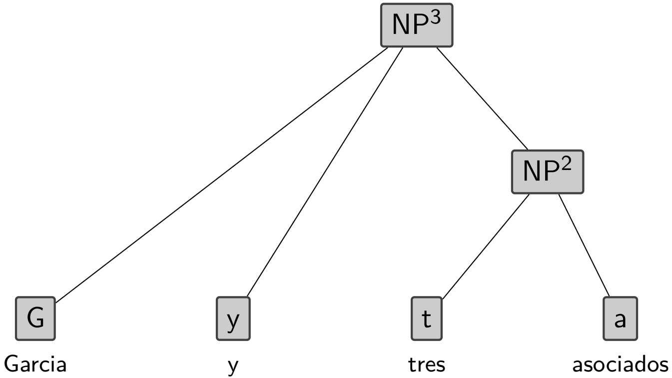
$\Gamma = \{G, y, t, a\}$



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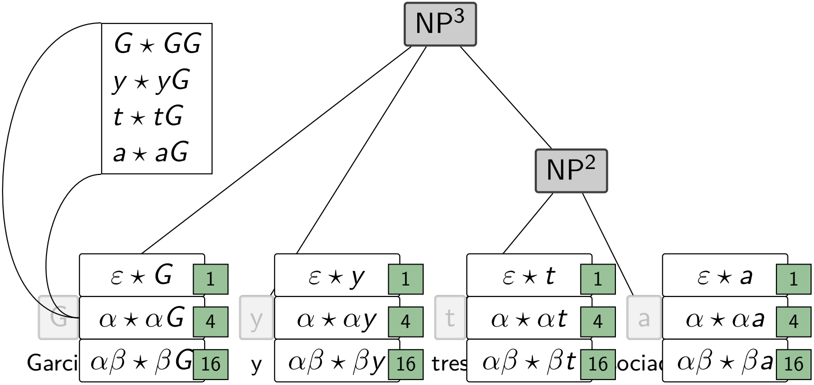

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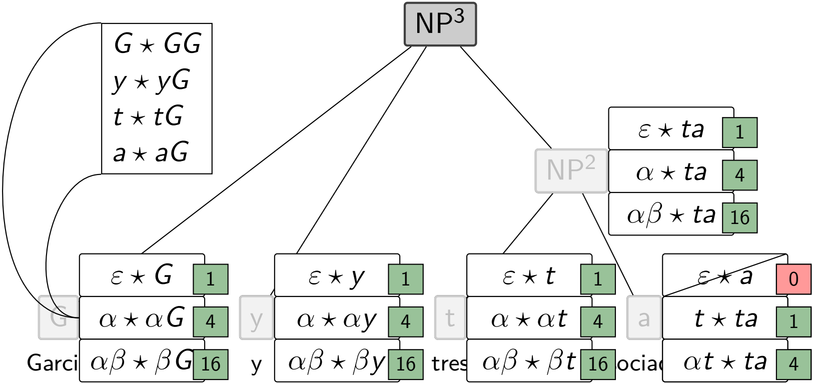

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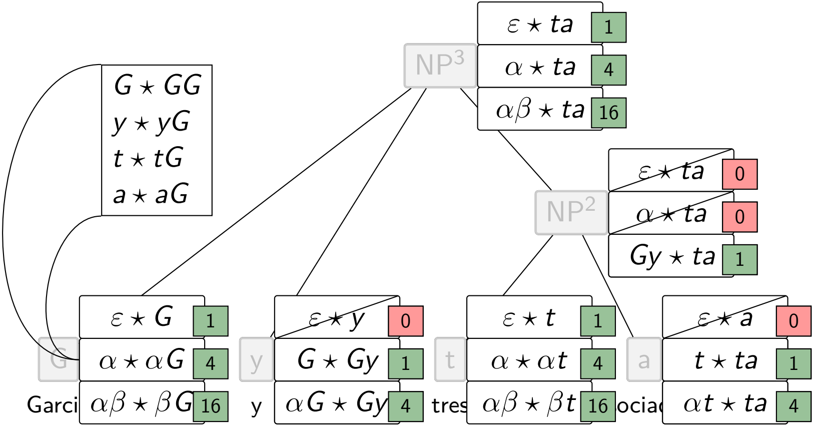

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