Automata and Logic for Concurrent Systems

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Laboratoire Spécification et Vérification

Workshop *Automaten und Logik*
Theorietag *Automaten und Formale Sprachen*
25.-27. September 2013, Ilmenau

What is a concurrent system?

- Collection of autonomous computing entities (processes) connected by some communication medium

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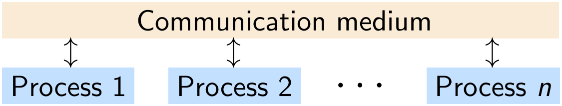
- Schematic view:



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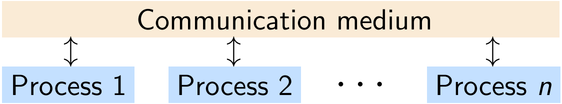

- Purpose:
  - entities collaborate on a task:
    terminating computation with input and output
  - entities model a reactive system:
    focus on behavior, properties of performed action sequence
    (e.g., mutual exclusion)

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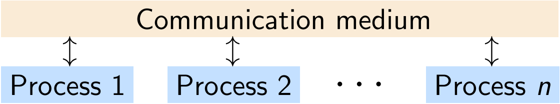

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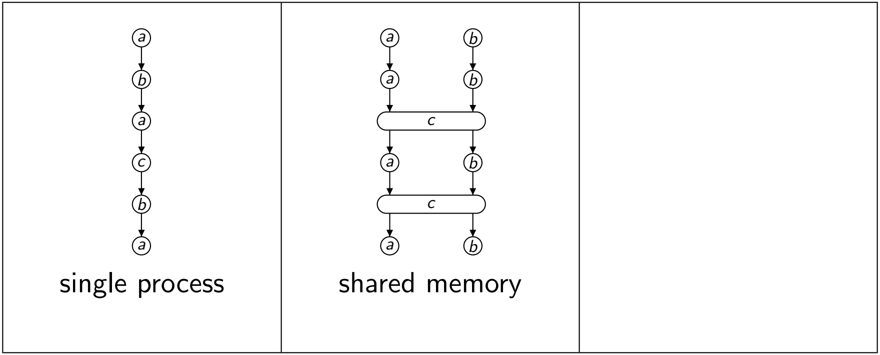
    (e.g., mutual exclusion)
- In this talk: formal modeling of concurrent reactive systems (in terms of automata) to make them accessible to formal methods

2. Classification

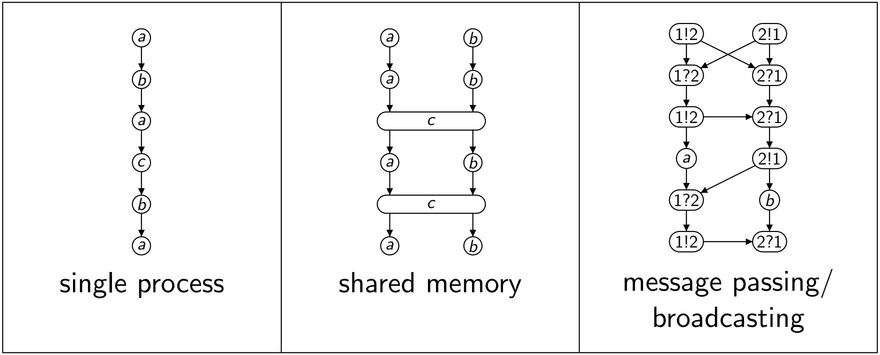
Form of communication

single process

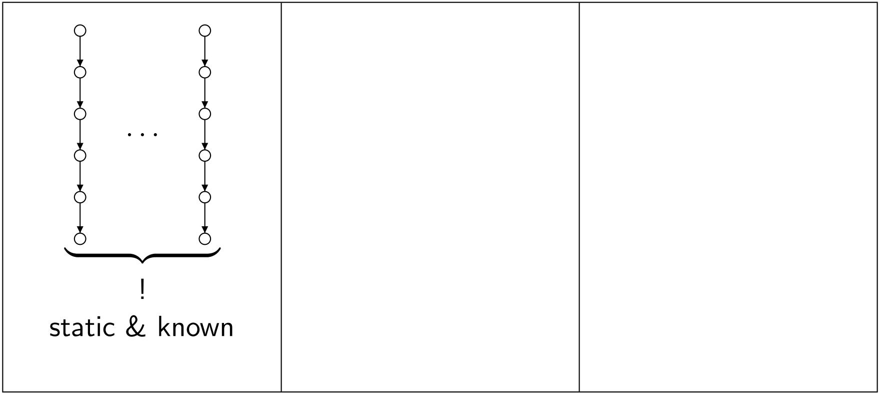
Form of communication



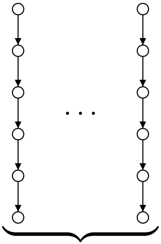
Form of communication



System architecture

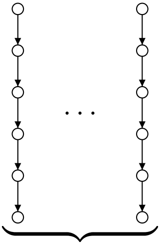


System architecture



!

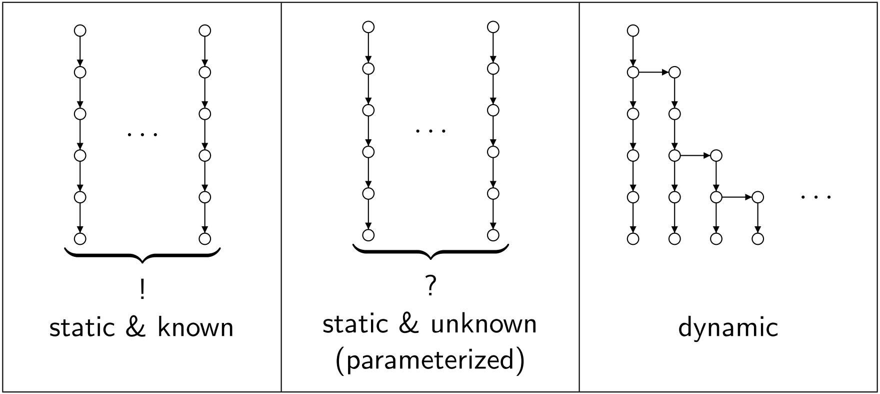
static & known



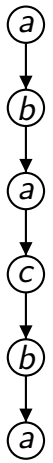
?

static & unknown
(parameterized)

System architecture

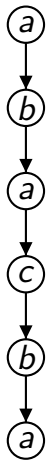


Type of single process



finite-state

Type of single process



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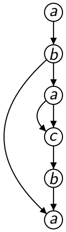
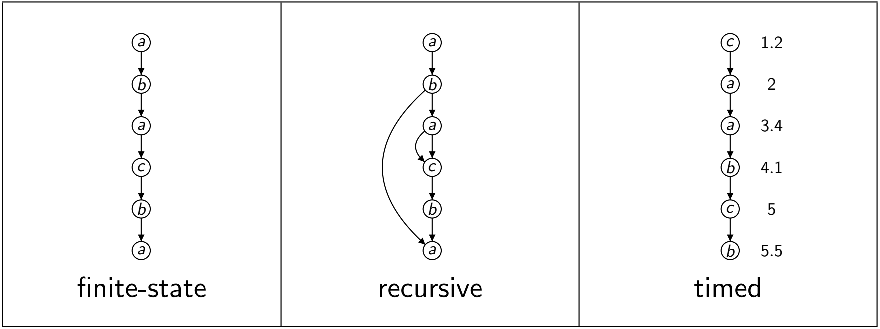
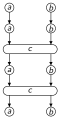
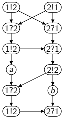
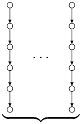
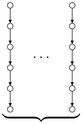
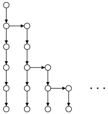
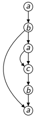
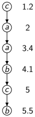


recursive

Type of single process



The various settings ...

 single process	 shared memory	 message passing/ broadcasting
 ! static & known	 ? static & unknown (parameterized)	 dynamic
 finite-state	 recursive	 timed

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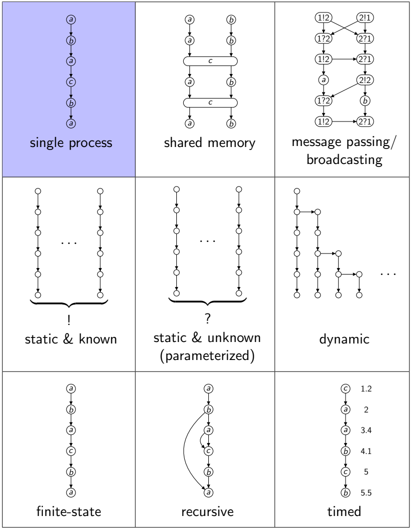

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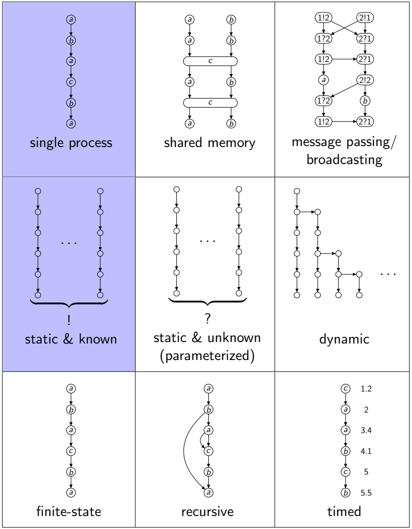

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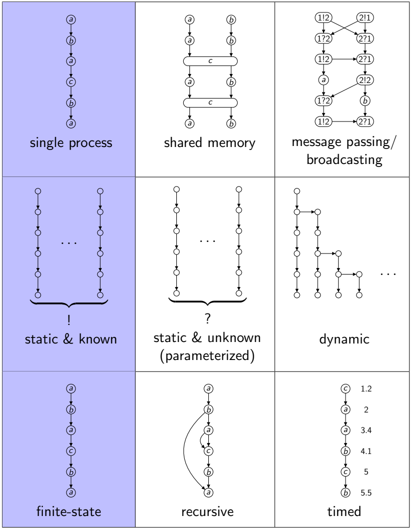

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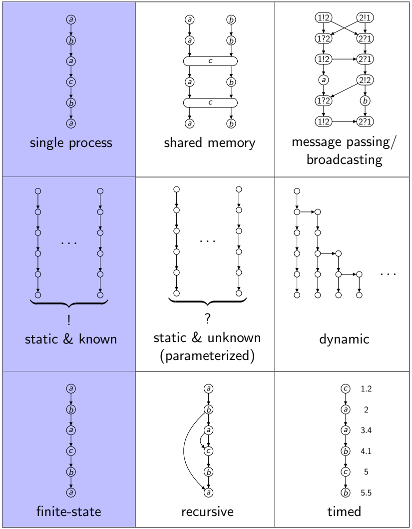
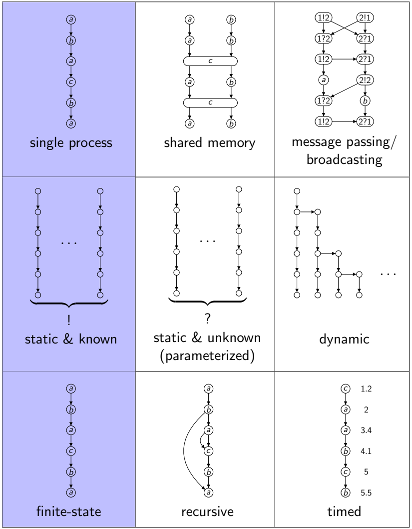


Behavior

- Words

The various settings ...



Behavior

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System model

- Finite automata
- Kripke structures

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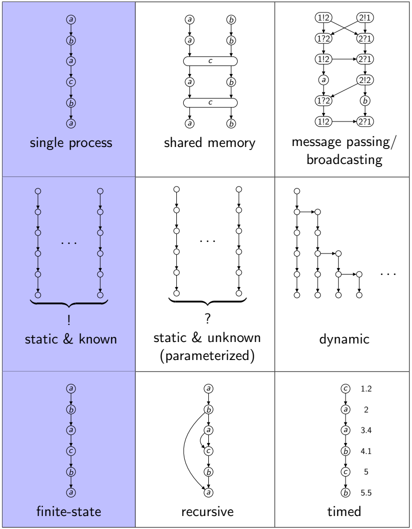

Behavior

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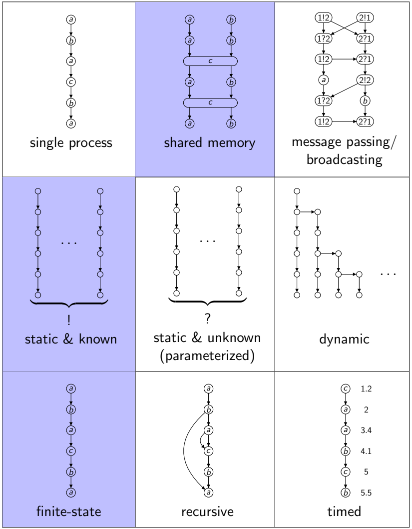
System model

- Finite automata
- Kripke structures

Specification

- Linear-time temporal logic (LTL)
- Monadic second-order logic (MSO)
- Regular expressions

The various settings ...



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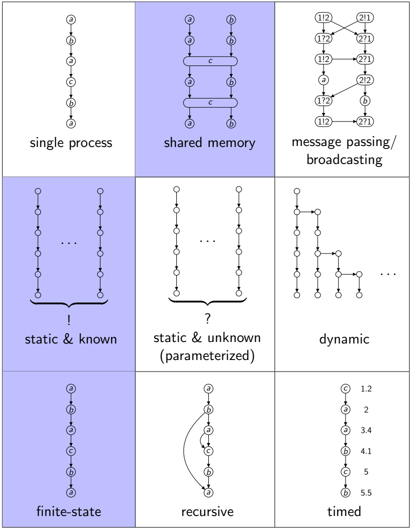

Behavior

- Mazurkiewicz traces
  [Mazurkiewicz '86]

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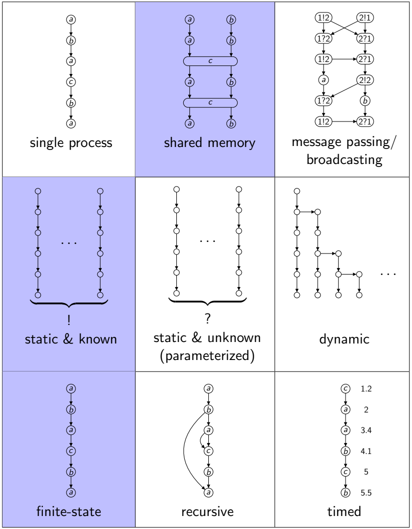

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- Asynchronous automata
  [Zielonka '87]
- Asynchronous cellular automata

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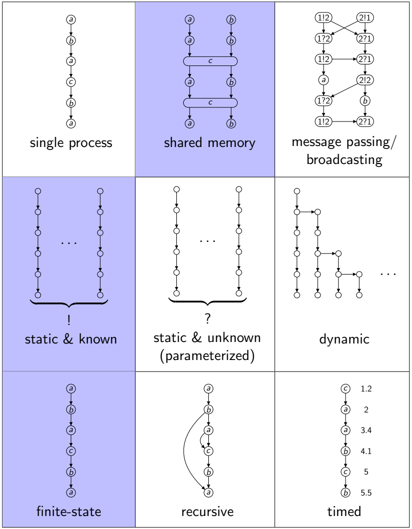

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- Monadic second-order logic (MSO)
- Regular (rational) expressions

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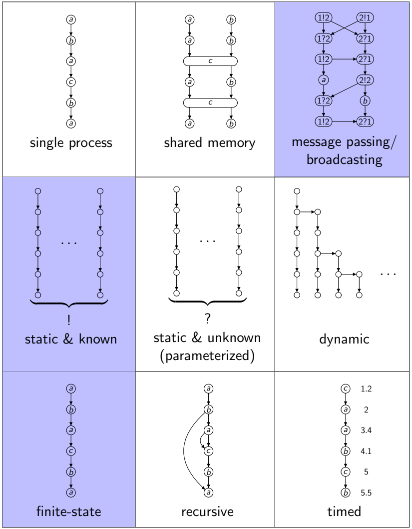

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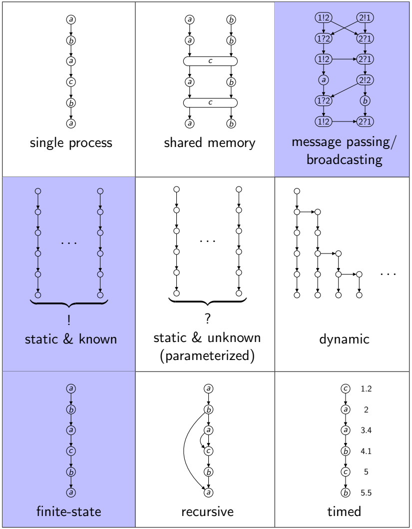
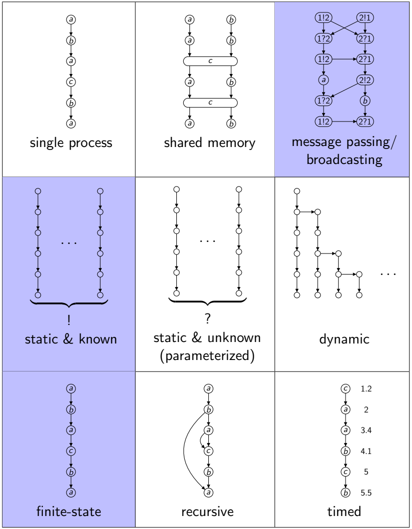


Behavior

- Message sequence charts

The various settings ...



Behavior

- Message sequence charts

System model

- Communicating automata
  [Brand-Zafiropulo '83]
- Lossy channel systems
  [Finkel '87, Abdulla-Jonsson '96]

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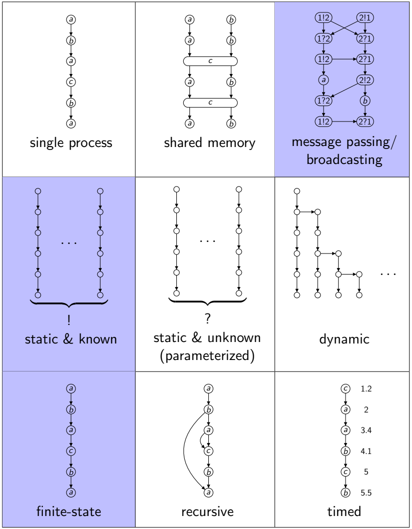

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Specification

- Temporal logic
- Monadic second-order logic (MSO)
- High-level expressions

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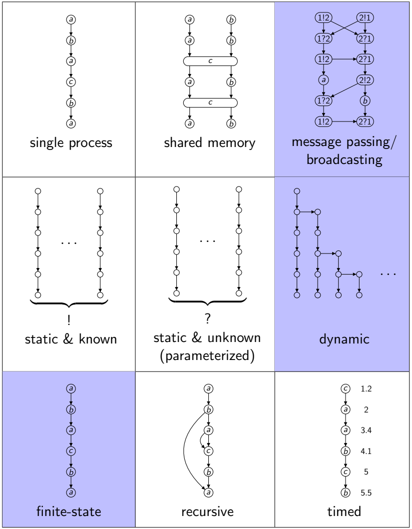

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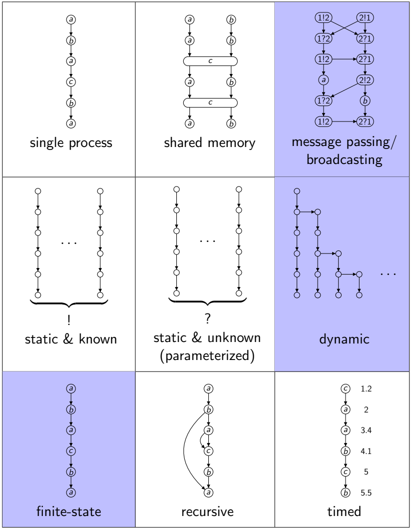
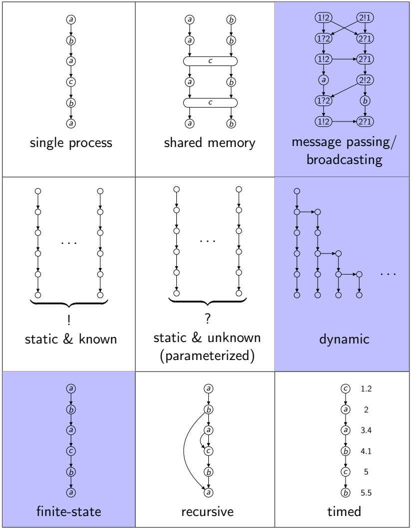


Behavior

- Dynamic message sequence charts

The various settings ...



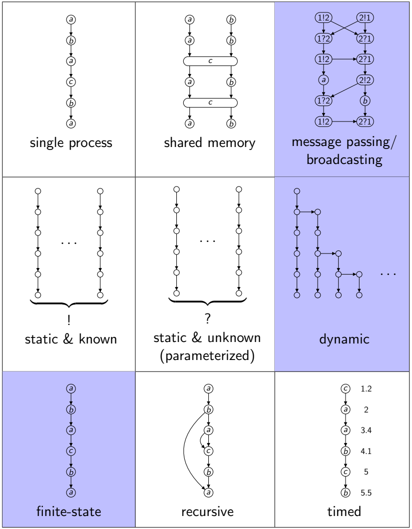
Behavior

- Dynamic message sequence charts

System model

- Dynamic communicating automata
  [B., Cyriac, Hélouët, Kara, Schwentick ’13]

The various settings ...



Behavior

- Dynamic message sequence charts

System model

- Dynamic communicating automata
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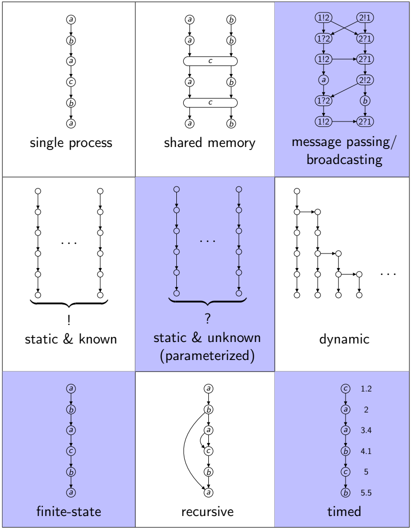
Specification

- High-level expressions with registers

The various settings ...



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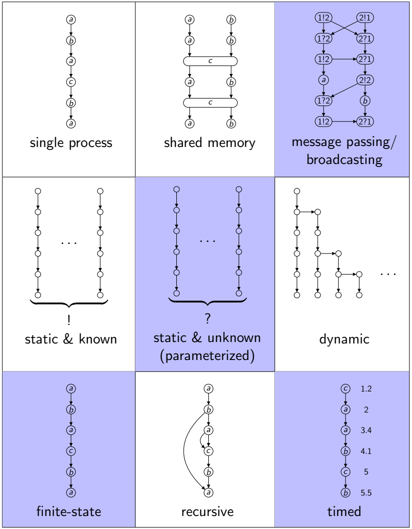
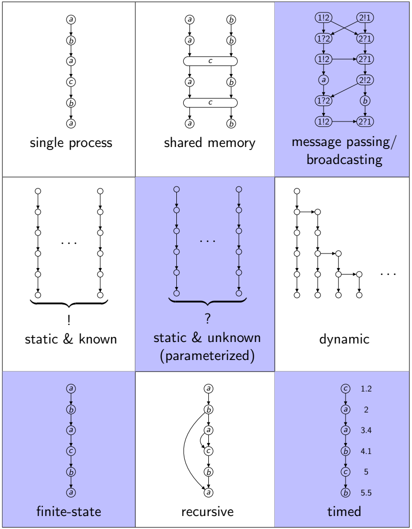


Behavior

- Words ?

The various settings ...



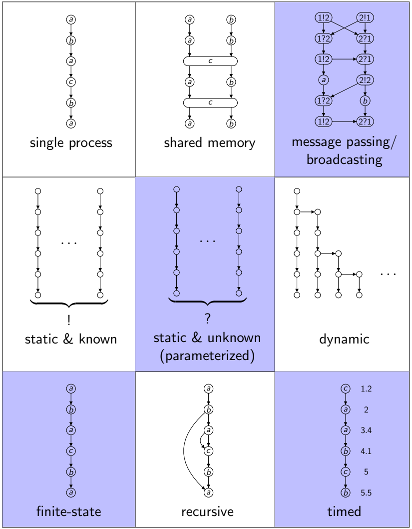
Behavior

- Words ?

System model

- Parametric ad-hoc networks
  [Delzanno-Sangnier et al. '10–'13]

The various settings ...



Behavior

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System model

- Parametric ad-hoc networks
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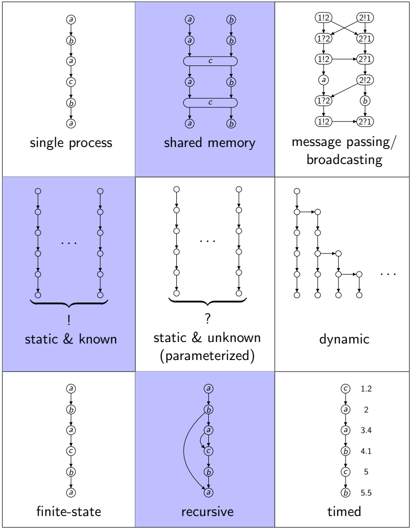
Specification

- Reachability questions

The various settings ...



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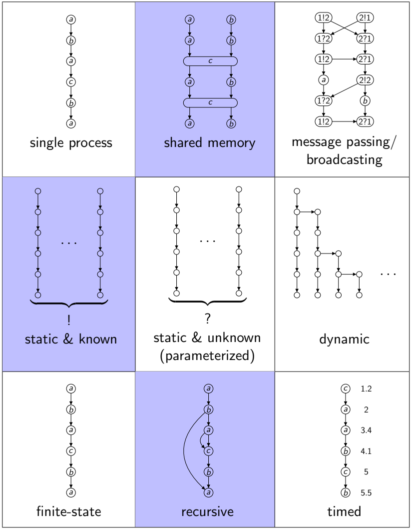
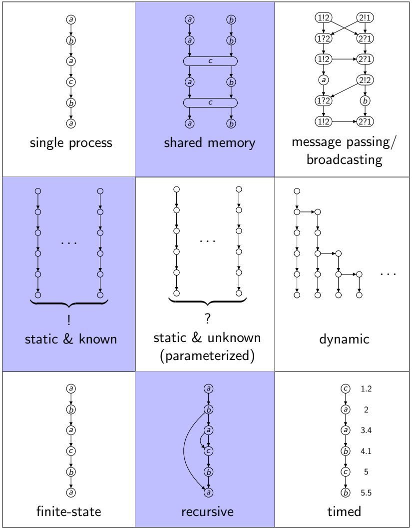


Behavior

- Nested traces

The various settings ...



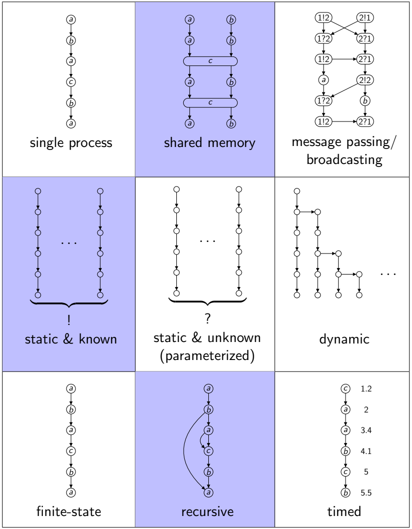
Behavior

- Nested traces

System model

- Multi-stack systems
  [La Torre et al. '07–'13], [Atig et al.]
- Nested-word automata
  [Alur et al. '04]

The various settings ...



Behavior

- Nested traces

System model

- Multi-stack systems
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Specification

- Temporal logic (such as LTL)
- Monadic second-order logic (MSO)
- Regular (rational) expressions

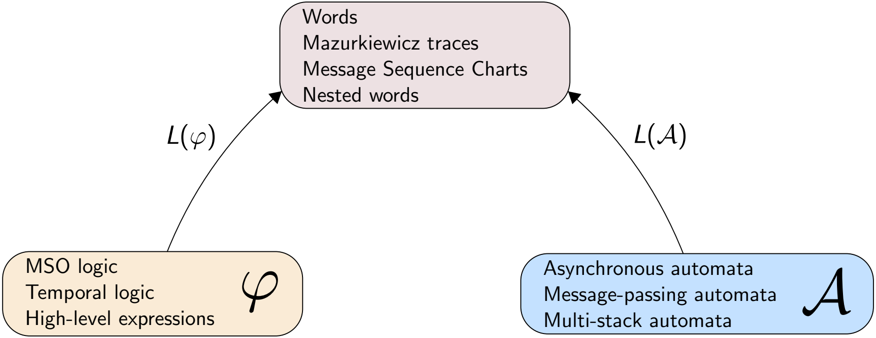
Landscape and Objectives

Words
Mazurkiewicz traces
Message Sequence Charts
Nested words

MSO logic
Temporal logic
High-level expressions
φ

Asynchronous automata
Message-passing automata
Multi-stack automata
$\mathcal{A}$

Landscape and Objectives



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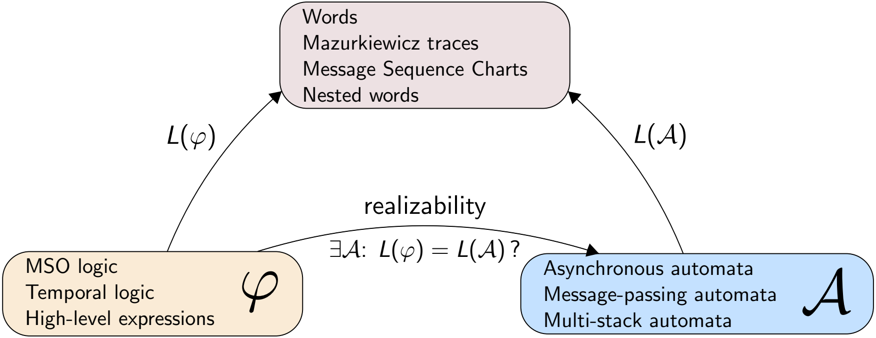

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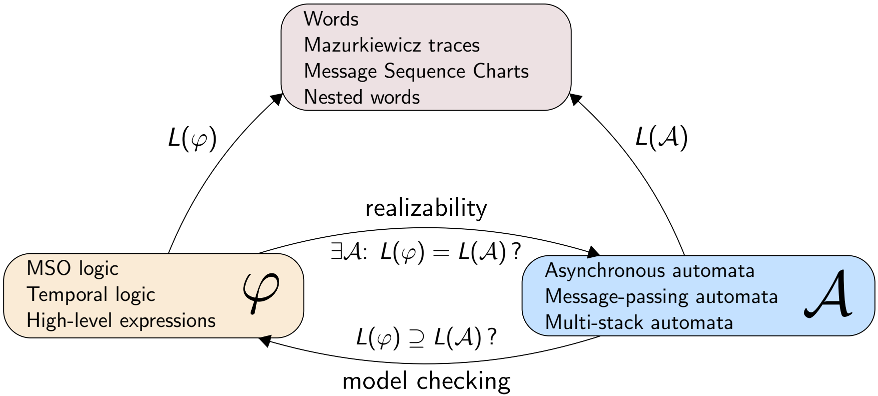

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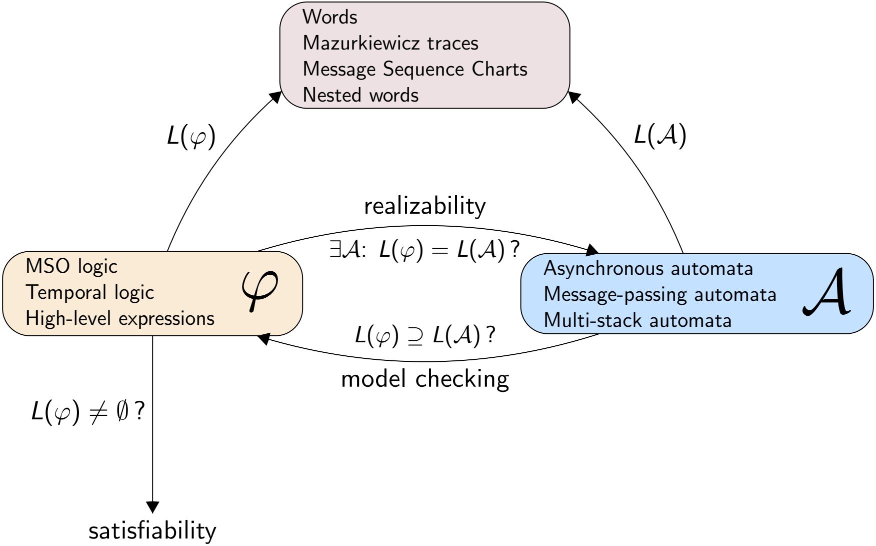
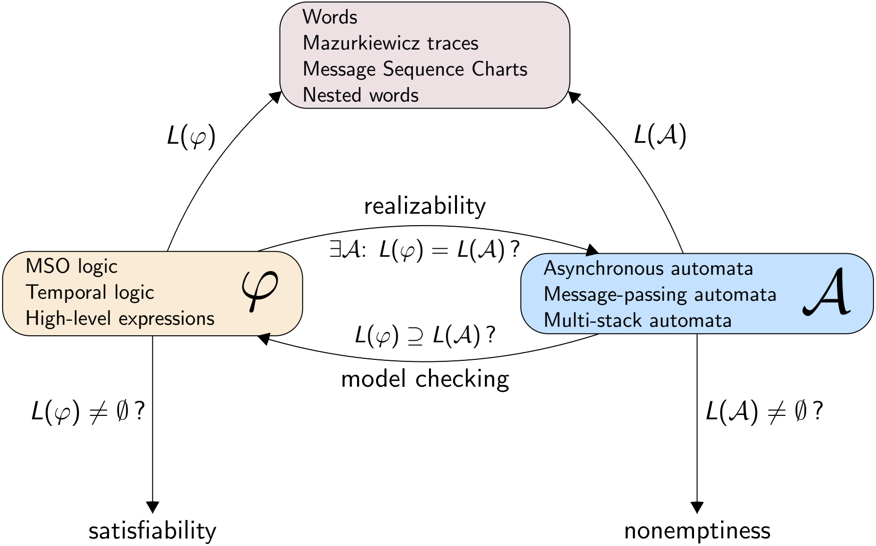
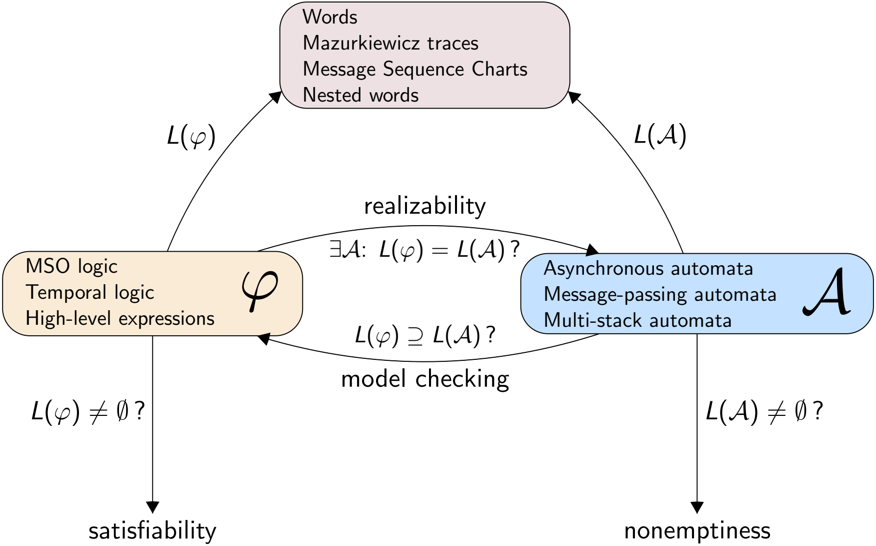


Landscape and Objectives



Landscape and Objectives: Linear-Time Setting



In this talk:

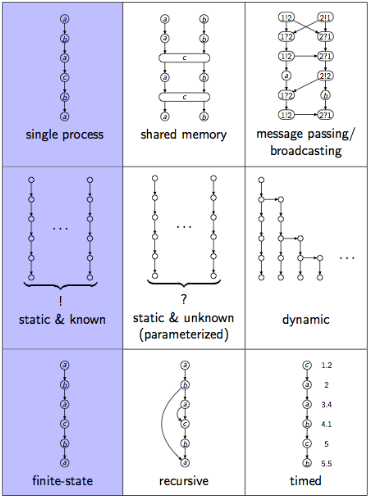
- Finite-State Sequential Systems
- Finite-State Shared-Memory Systems
- Recursive Shared-Memory Systems
- Message-Passing Systems

In this talk:

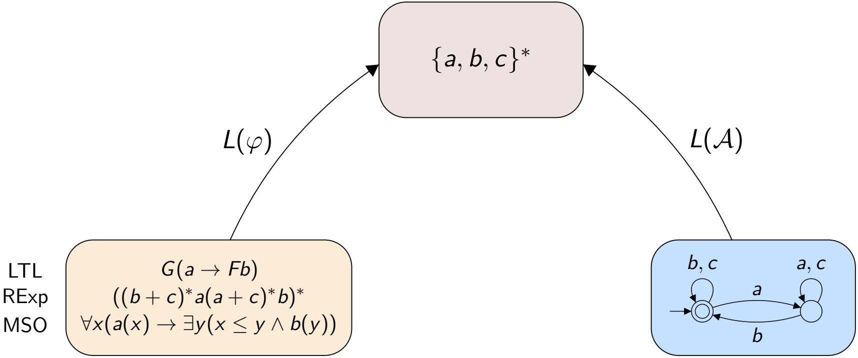
- Finite-State Sequential Systems
- Finite-State Shared-Memory Systems
- Recursive Shared-Memory Systems
- Message-Passing Systems

with static and known system architecture

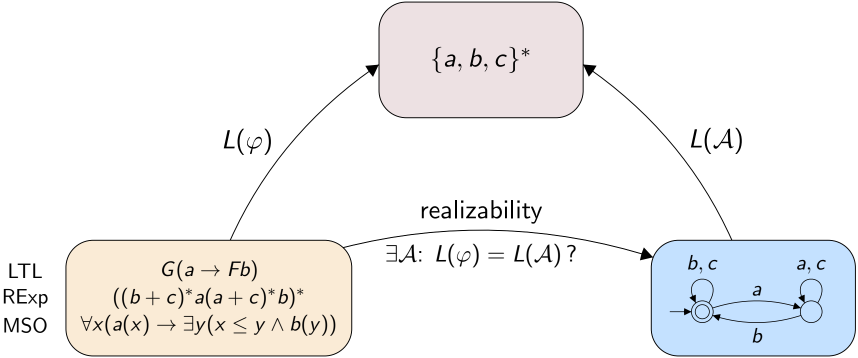
3. Finite-State Sequential Systems



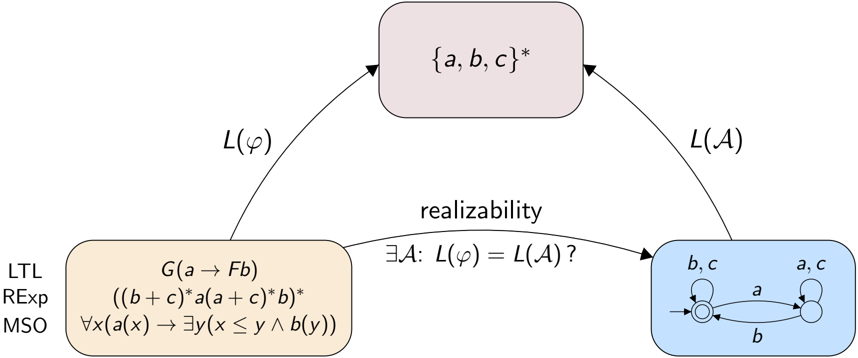
Finite-State Sequential Systems



Finite-State Sequential Systems



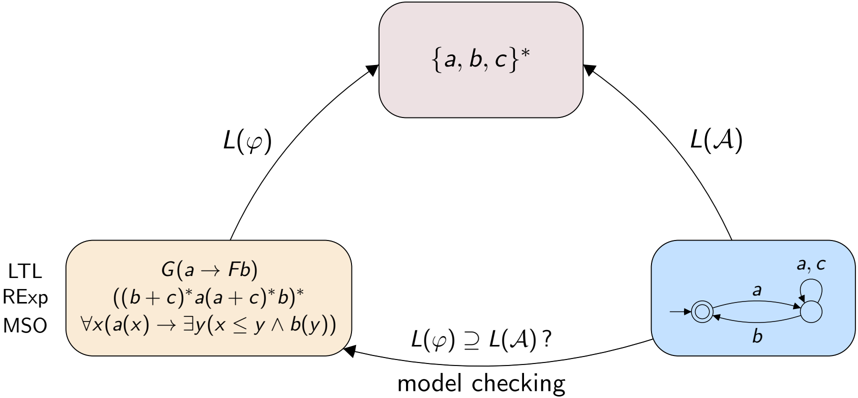
Finite-State Sequential Systems



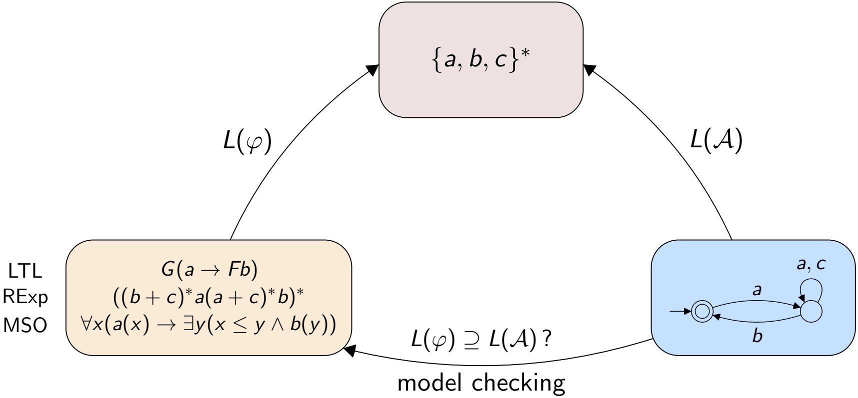
Theorem (Büchi-Elgot-Trakhtenbrot ’60s)

Every MSO formula is equivalent to some (deterministic) finite automaton.

Finite-State Sequential Systems



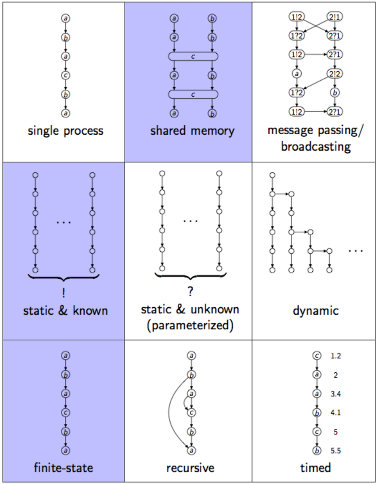
Finite-State Sequential Systems



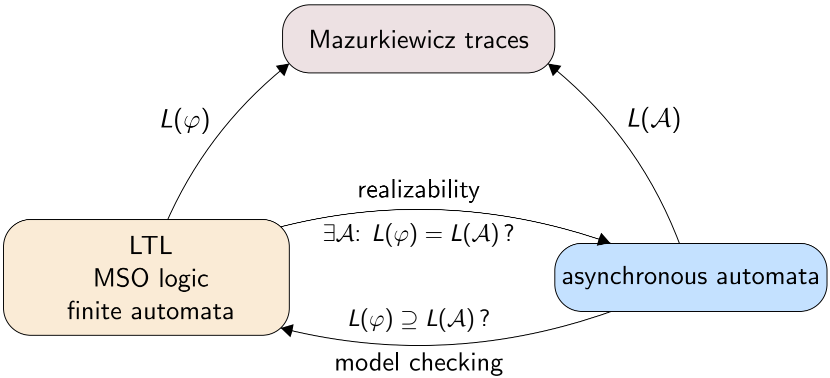
Theorem (Büchi-Elgot-Trakhtenbrot '60s; Sistla-Clarke '85)

Model checking against MSO is decidable, but nonelementary.
Model checking LTL is PSPACE-complete.

4. Finite-State Shared-Memory Systems



Finite-State Shared-Memory Systems



Asynchronous Automata and Mazurkiewicz Traces

$Proc = \{1, 2\}$ $\Sigma_1 = \{a_1, b_1, c\}$ $\Sigma_2 = \{a_2, b_2, c\}$

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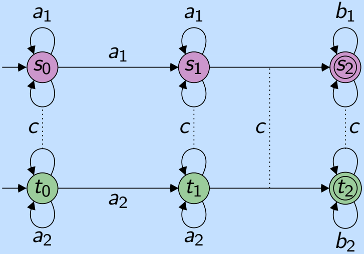
Asynchronous Automaton

$$(s_0) \xrightarrow{a_1} (s_1)$$
$$(s_0, t_0) \xrightarrow{c} (s_0, t_0)$$
$$(s_1, t_1) \xrightarrow{c} (s_2, t_2)$$
$$(s_0, t_1) \not\xrightarrow{c} (s_0, t_1)$$



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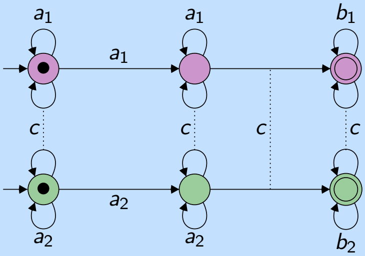

Mazurkiewicz Trace

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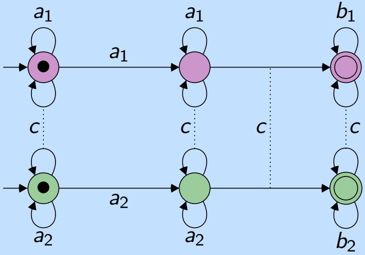

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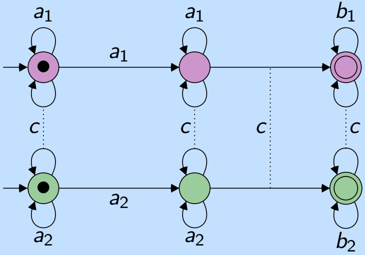

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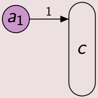

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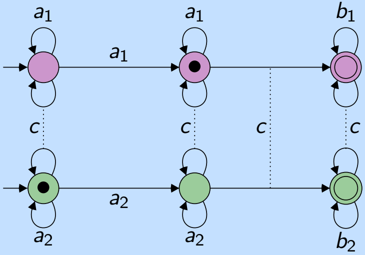

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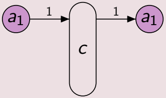

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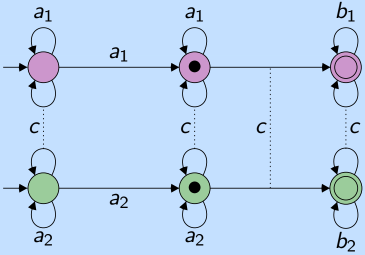

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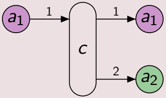

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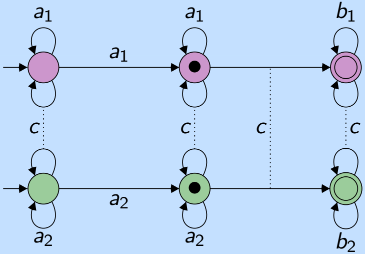

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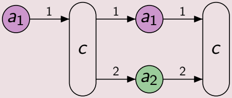

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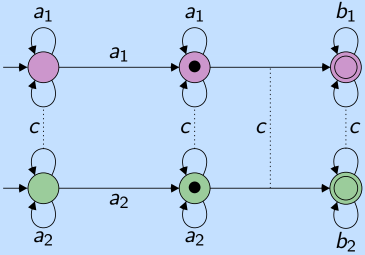

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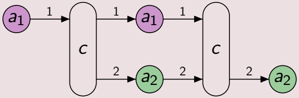

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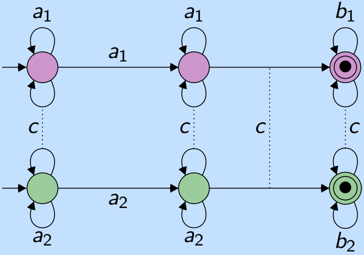

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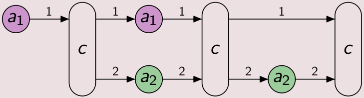

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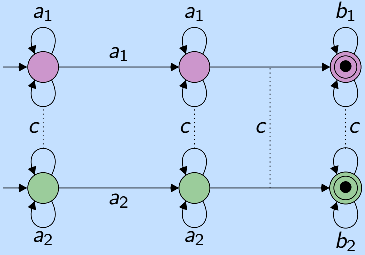

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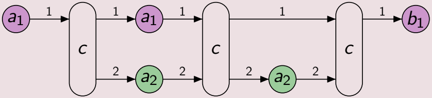

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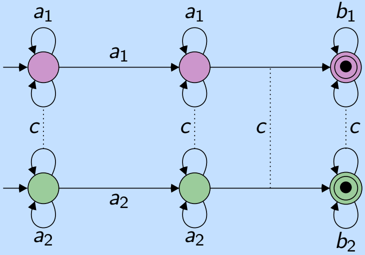

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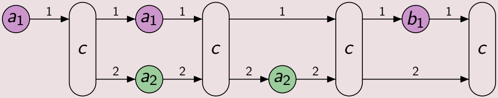

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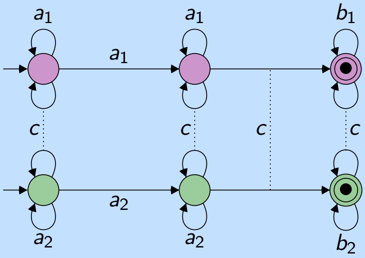

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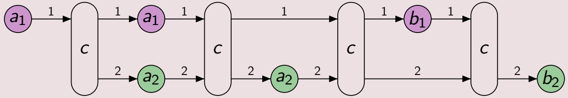

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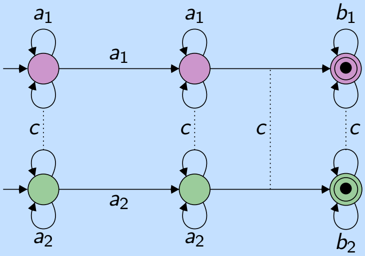

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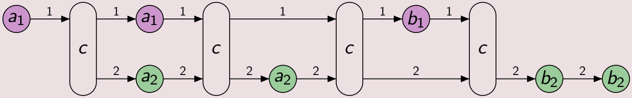

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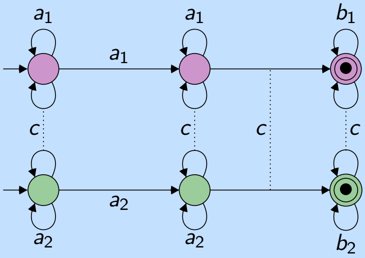

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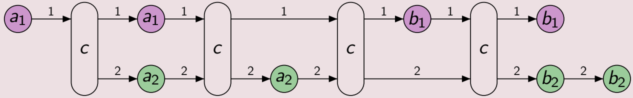
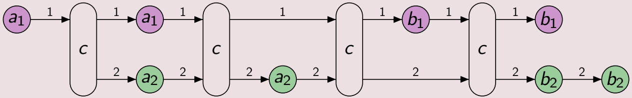


Mazurkiewicz Traces and Their Linearizations

Mazurkiewicz Trace $\quad t = (E, \rightarrow_1, \rightarrow_2, \lambda) \quad \lambda : E \rightarrow \Sigma \stackrel{\text{def}}{=} \Sigma_1 \cup \Sigma_2$



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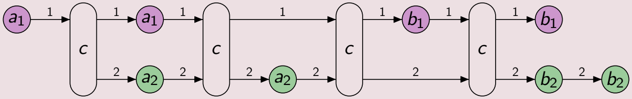
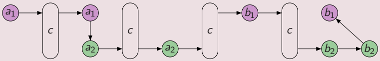


Linearizations $\quad w \in Lin(t) \subseteq \Sigma^* \quad \rightsquigarrow \quad trace(w) = t$



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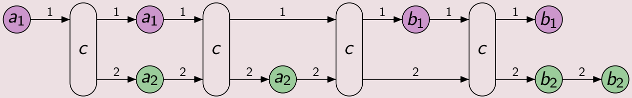

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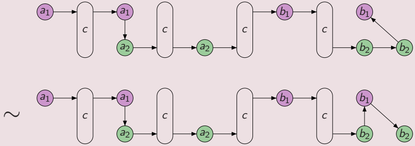

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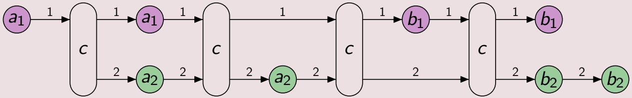

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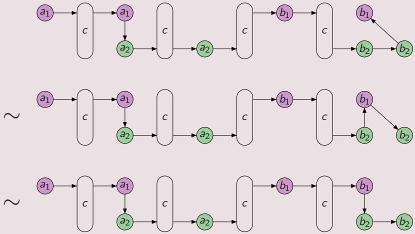

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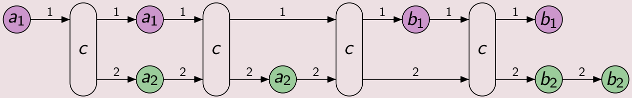

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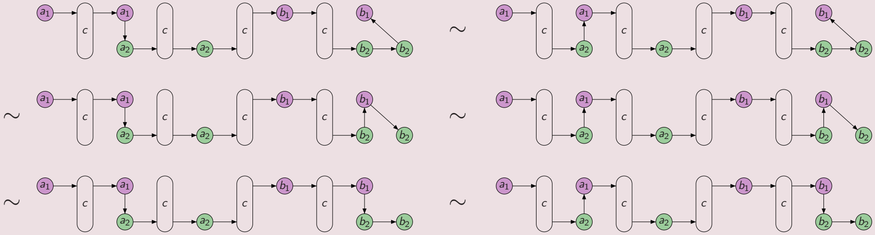
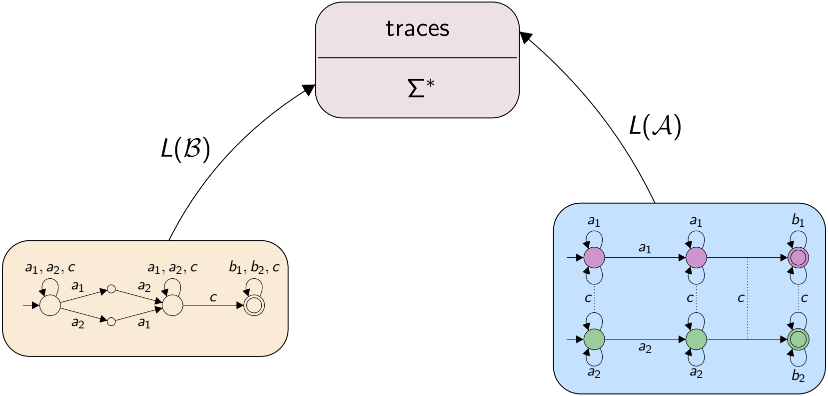


Finite-State Shared-Memory Systems



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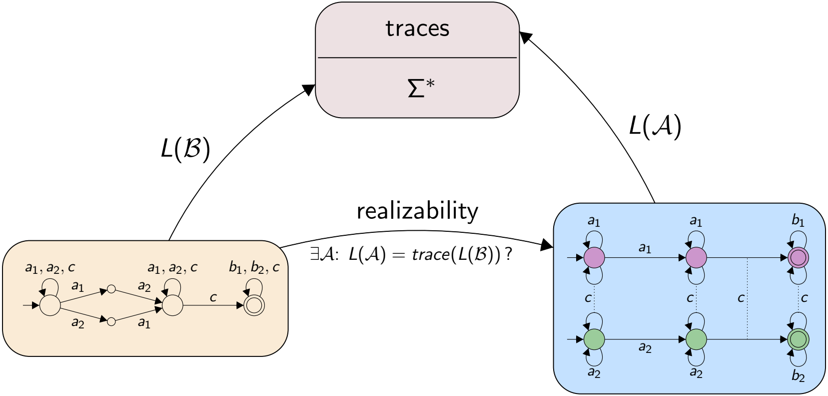

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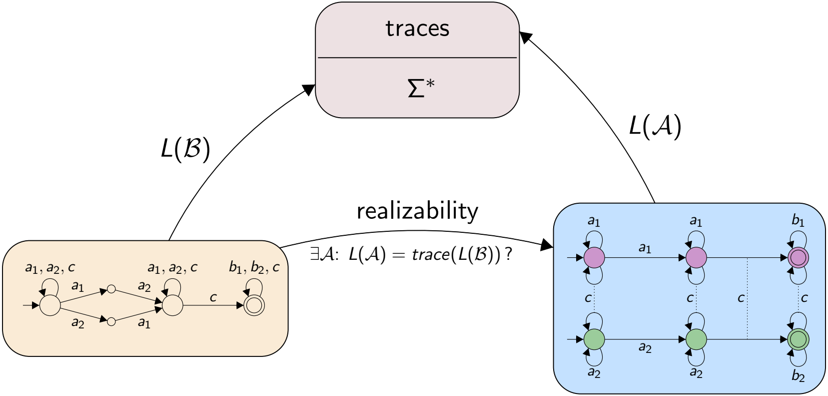
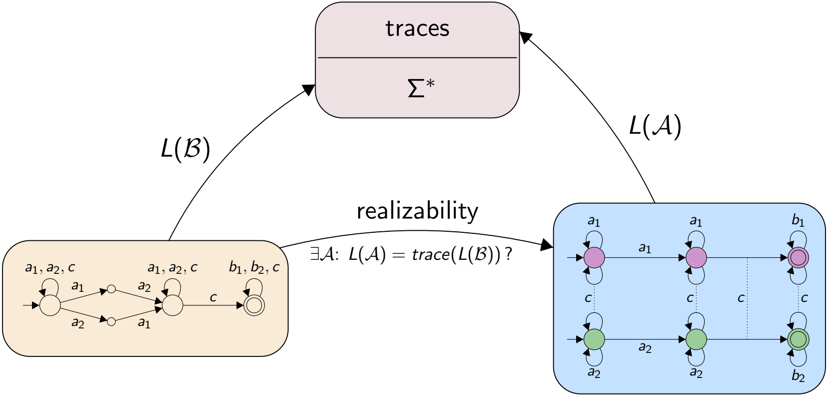


Theorem (Sakarovitch '92)

Realizability for regular specifications is undecidable.

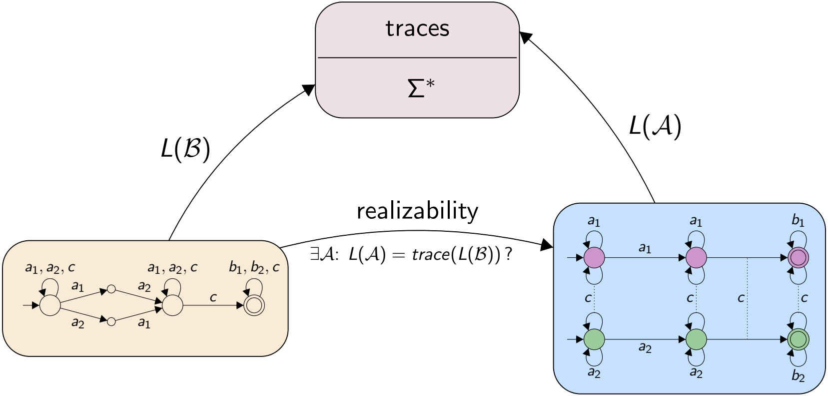
Finite-State Shared-Memory Systems



Theorem (Zielonka '87)

Let $L \subseteq \Sigma^*$ be a $\sim$-closed regular language. There is a (deterministic) asynchronous automaton $\mathcal{A}$ such that $L(\mathcal{A}) = trace(L)$.

Finite-State Shared-Memory Systems



Theorem (Muscholl '94, Peled-Wilke-Wolper '98)

It is decidable (PSPACE-complete) if the language of a finite automaton is $\sim$-closed (PTIME for deterministic automata).

Monadic Second-Order Logic

Monadic Second-Order Logic (MSO)

- $x \rightarrow_p y$ — x and y are successive events on process $p \in Proc$

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- $x \in X$ — event x is contained in set of events X

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▸ $x = y$	
▸ $x \in X$	event x is contained in set of events X
▸ $\exists x \varphi$	there is event x such that φ

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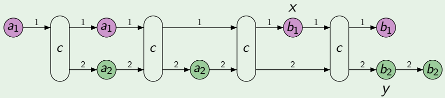
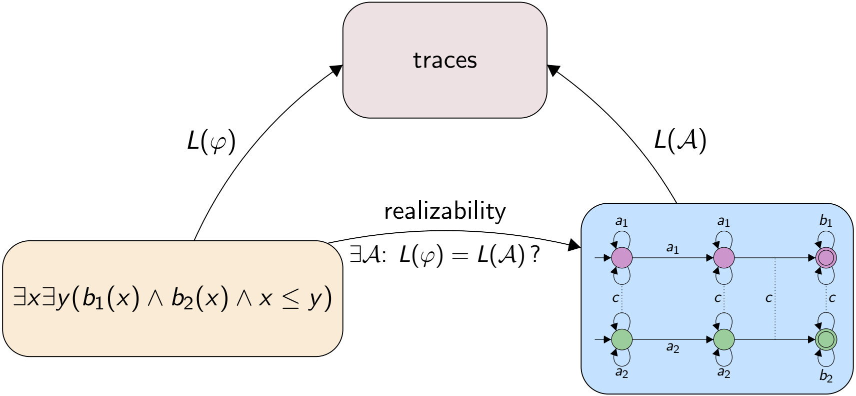
Example



$\models \; \exists x \exists y (b_1(x) \wedge b_2(x) \wedge x \leq y)$

where $\leq = (\rightarrow_1 \cup \rightarrow_2)^*$

Finite-State Shared-Memory Systems



Theorem (Thomas '90)

MSO logic and asynchronous automata are expressively equivalent.

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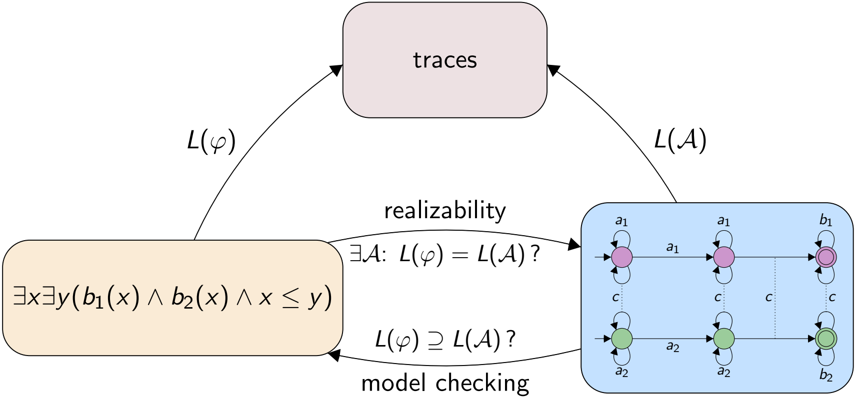

Theorem (Thomas '90)

MSO logic and asynchronous automata are expressively equivalent.

⇨ MSO model checking is decidable.

Global Temporal Logic

Global Temporal Logic

$\mathsf{LTrL}_{\forall} \qquad \varphi ::= \mathtt{tt} \mid \langle a \rangle \varphi \mid \varphi_1 \, \mathsf{U}_{\forall} \, \varphi_2 \mid \neg \varphi \mid \varphi_1 \vee \varphi_2 \qquad a \in \Sigma$

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$\mathrm{LTrL}_\forall$	$\varphi ::= \mathtt{tt} \mid \langle a \rangle \varphi \mid \varphi_1 \, \mathsf{U}_\forall \, \varphi_2 \mid \neg\varphi \mid \varphi_1 \vee \varphi_2$	$a \in \Sigma$
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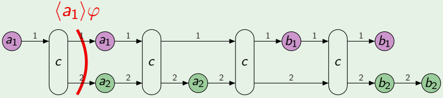
Semantics



$\models \ \langle a_1 \rangle \varphi$

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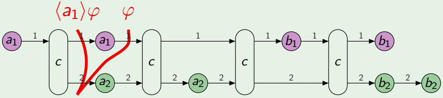

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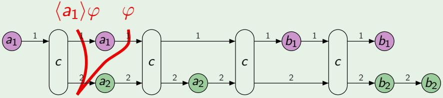

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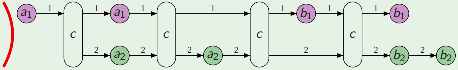

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Global Temporal Logic

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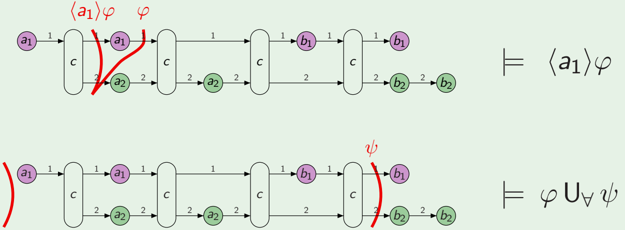
LTrL$_\forall$	$\varphi ::= \mathtt{tt} \mid \langle a \rangle \varphi \mid \varphi_1 \, \mathsf{U}_\forall \, \varphi_2 \mid \neg \varphi \mid \varphi_1 \vee \varphi_2$	$a \in \Sigma$
LTrL$_\exists$	$\varphi ::= \qquad\qquad \mathsf{U}_\exists$	

Semantics



Global Temporal Logic

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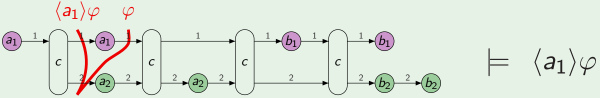
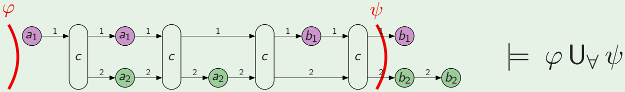
$\mathsf{LTrL}_\forall$ $\quad \varphi ::= \mathtt{tt} \mid \langle a\rangle\varphi \mid \varphi_1 \,\mathsf{U}_\forall\, \varphi_2 \mid \neg\varphi \mid \varphi_1 \vee \varphi_2 \qquad a \in \Sigma$

$\mathsf{LTrL}_\exists$ $\quad \varphi ::= \qquad \mathsf{U}_\exists$

Semantics





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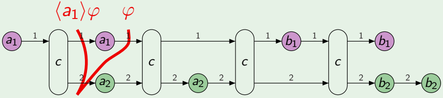
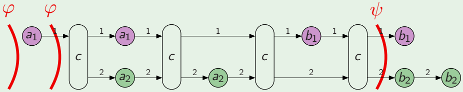
Semantics



$\models \ \langle a_1 \rangle \varphi$



$\models \ \varphi \, \mathsf{U}_\forall \, \psi$

Global Temporal Logic

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$$\text{LTrL}_\forall \qquad \varphi ::= \mathtt{tt} \mid \langle a\rangle\varphi \mid \varphi_1 \,\mathsf{U}_\forall\, \varphi_2 \mid \neg\varphi \mid \varphi_1 \vee \varphi_2 \qquad a \in \Sigma$$

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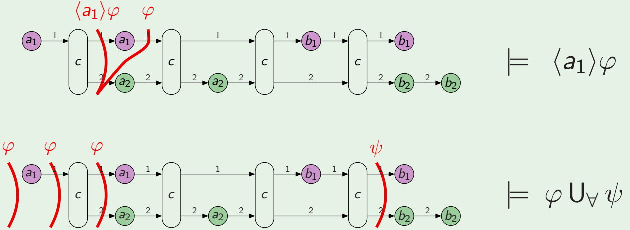
Semantics



Global Temporal Logic

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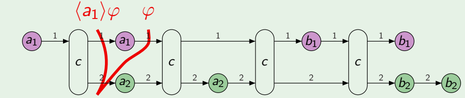
Semantics



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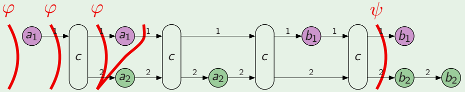

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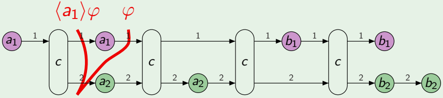

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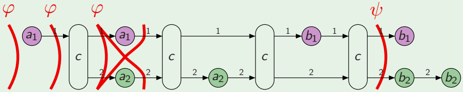

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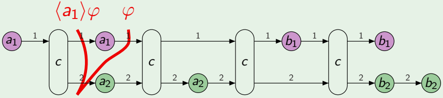
Semantics



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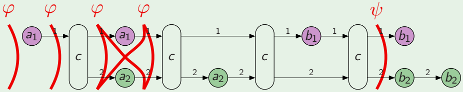

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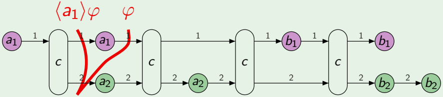

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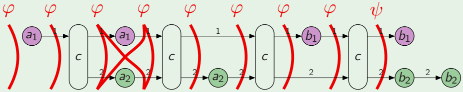

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Global Temporal Logic

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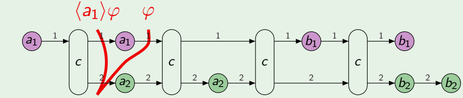

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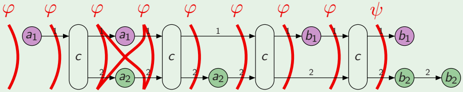


$\models \quad \texttt{tt}\,\mathsf{U}_\forall\,\langle b_1\rangle\langle b_2\rangle\texttt{tt}$

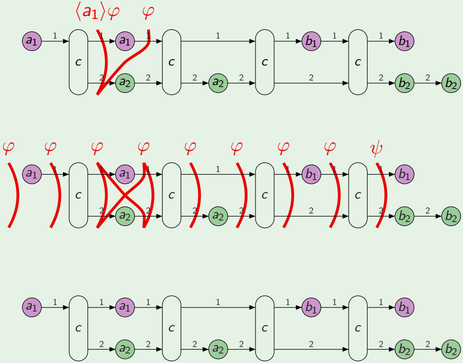
Global Temporal Logic

Global Temporal Logic

$\mathrm{LTrL}_\forall \qquad \varphi ::= \mathtt{tt} \mid \langle a \rangle \varphi \mid \varphi_1 \,\mathsf{U}_\forall\, \varphi_2 \mid \neg\varphi \mid \varphi_1 \vee \varphi_2 \qquad a \in \Sigma$

$\mathrm{LTrL}_\exists \qquad \varphi ::= \qquad \mathsf{U}_\exists$

Semantics



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Global Temporal Logic

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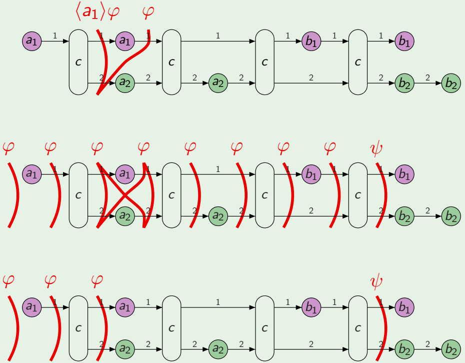
Semantics



$\models \langle a_1\rangle\varphi$

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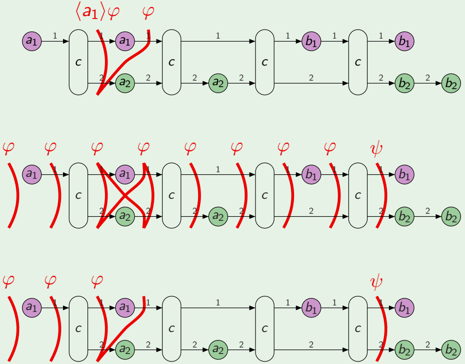

$\models \langle a_1 \rangle \varphi$

$\models \mathtt{tt} \, \mathsf{U}_\forall \, \langle b_1 \rangle \langle b_2 \rangle \mathtt{tt}$

$\models \varphi \, \mathsf{U}_\exists \, \psi$

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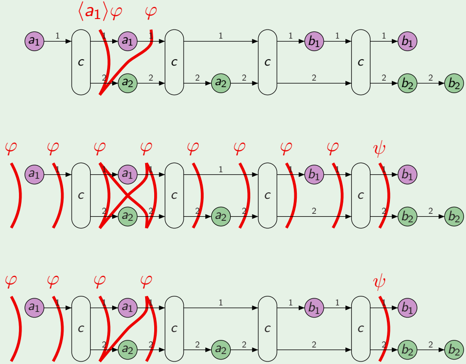
Semantics



$\models \; \langle a_1\rangle\varphi$

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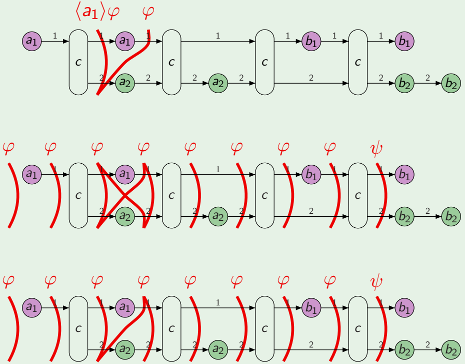
Semantics

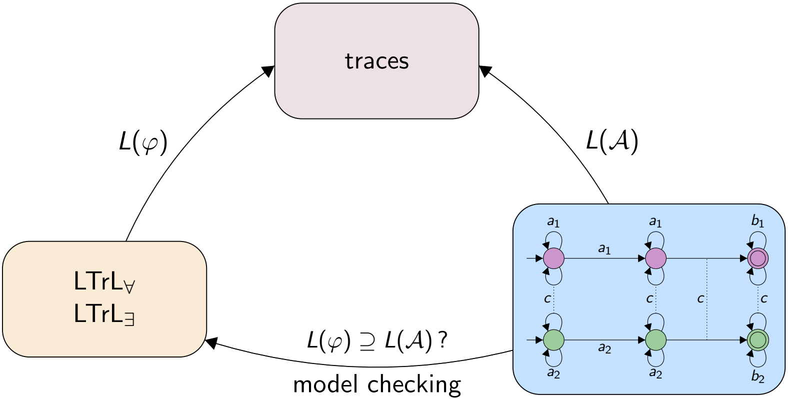


$\models \;\langle a_1\rangle\varphi$

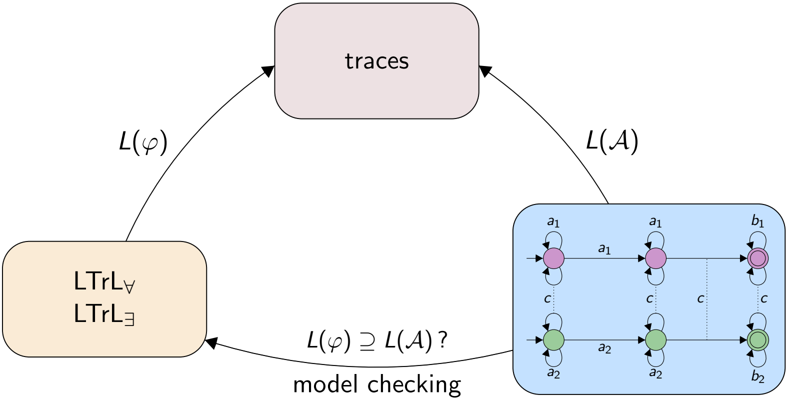
$\models \;\mathtt{tt}\,\mathsf{U}_\forall\,\langle b_1\rangle\langle b_2\rangle\mathtt{tt}$

$\models \;\varphi\,\mathsf{U}_\exists\,\psi$

Finite-State Shared-Memory Systems

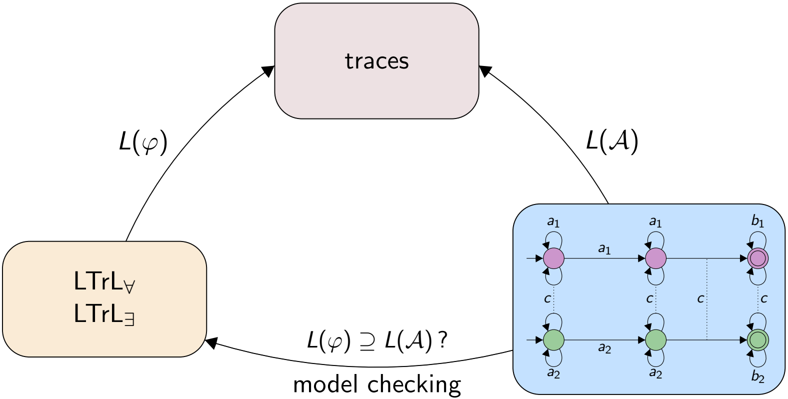


Finite-State Shared-Memory Systems



Theorem (Walukiewicz '98; Alur-McMillan-Peled '98)

- $\mathrm{LTrL}_\forall$ model checking is nonelementary.



Theorem (Walukiewicz '98; Alur-McMillan-Peled '98)

- $\mathrm{LTrL}_\forall$ model checking is nonelementary.
- $\mathrm{LTrL}_\exists$ model checking is undecidable.

Local Temporal Logic

Local Temporal Logic

$$\varphi \quad ::= \quad a \mid \mathsf{EX}\varphi \mid \mathsf{EX}_p\varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2 \mid \varphi_1 \,\mathsf{U}_p\, \varphi_2 \mid \neg\varphi \mid \varphi_1 \vee \varphi_2$$

$$a \in \Sigma, p \in Proc$$

Local Temporal Logic

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$$a \in \Sigma, p \in Proc$$

Semantics (wrt. trace $t = (E, (\rightarrow_p)_{p \in Proc}, \lambda)$ and $e \in E$)

- $t, e \models \mathsf{EX}\varphi$ if there is $f \in E$ such that $e \lessdot f$ and $t, f \models \varphi$

Local Temporal Logic

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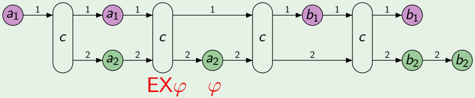

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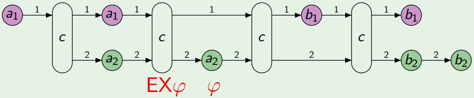

- $t, e \models \mathsf{EX}_p\varphi$ if there is $f \in E$ such that $e \leq f$ and $e \rightarrow_p f$ and $t, f \models \varphi$

Local Temporal Logic

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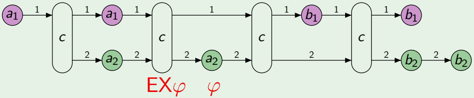

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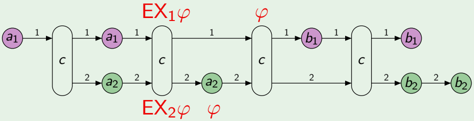

Temporal Logic

Semantics (wrt. trace $t = (E, (\rightarrow_p)_{p \in Proc}, \lambda)$ and $e \in E$)

- $t, e \models \overline{\mathsf{EX}}_p \varphi$ if there is $f \in E$ such that $\lambda(f) \in \Sigma_p$ and $t, f \models \varphi$ and f is the first p-event not below e wrt. $\leq$

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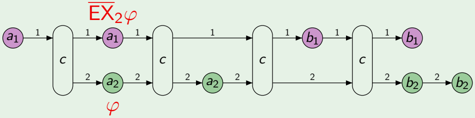

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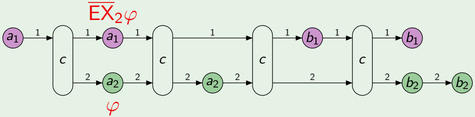

- $t, e \models \varphi \,\mathsf{U}\, \psi$ if there is $f \in E$ such that $t, f \models \psi$ and $t, e' \models \varphi$ for all $e' \in E$ with $e \leq e' < f$

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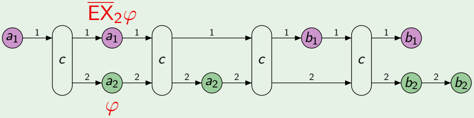

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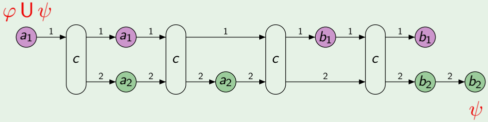

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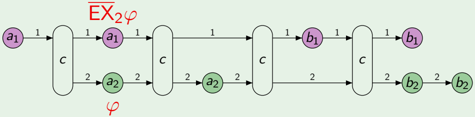

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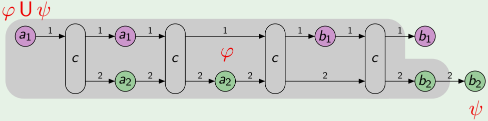

Temporal Logic

Observation (Gastin-Kuske '03)

All these modalities are MSO-definable!

Temporal Logic

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- $t, e \models \varphi \,\mathsf{U}\, \psi$ — if there is $f \in E$ such that $t, f \models \psi$ and $t, e' \models \varphi$ for all $e' \in E$ with $e \leq e' < f$

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- $t, e \models \varphi \,\mathsf{U}\, \psi$ if there is $f \in E$ such that $t, f \models \psi$ and $t, e' \models \varphi$ for all $e' \in E$ with $e \leq e' < f$

Example

- $\mathrm{MSO}^{\mathsf{EX}}(x, Y) = \exists y\,(y \in Y \wedge x \lessdot y)$

Temporal Logic

Observation (Gastin-Kuske '03)

All these modalities are MSO-definable!

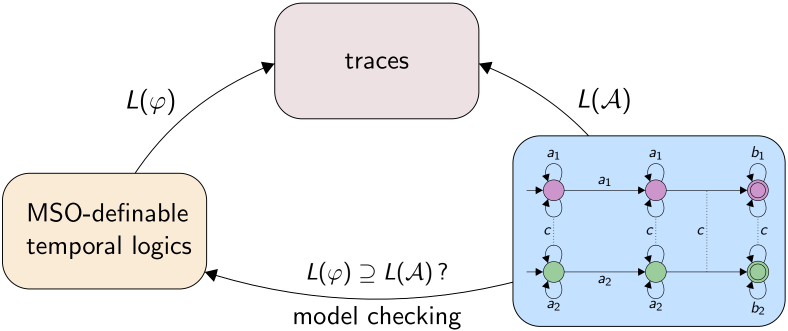
Semantics (wrt. trace $t = (E, (\rightarrow_p)_{p \in Proc}, \lambda)$ and $e \in E$)

- $t, e \models \mathsf{EX}\varphi$ if there is $f \in E$ such that $e \lessdot f$ and $t, f \models \varphi$
- $t, e \models \mathsf{EX}_p\varphi$ if there is $f \in E$ such that $e \rightarrow_p f$ and $t, f \models \varphi$ and f is the first p-event not below e wrt. $\leq$
- $t, e \models \varphi \,\mathsf{U}\, \psi$ if there is $f \in E$ such that $t, f \models \psi$ and $t, e' \models \varphi$ for all $e' \in E$ with $e \leq e' < f$

Example

- $\mathrm{MSO}^{\mathsf{EX}}(x, Y) = \exists y\,(y \in Y \wedge x \lessdot y)$
- $\mathrm{MSO}^{\mathsf{U}}(x, X, Y) = \exists y\,(y \in Y \wedge x \leq y \wedge \forall x'(x \leq x' < y \rightarrow x' \in X))$

Finite-State Shared-Memory Systems



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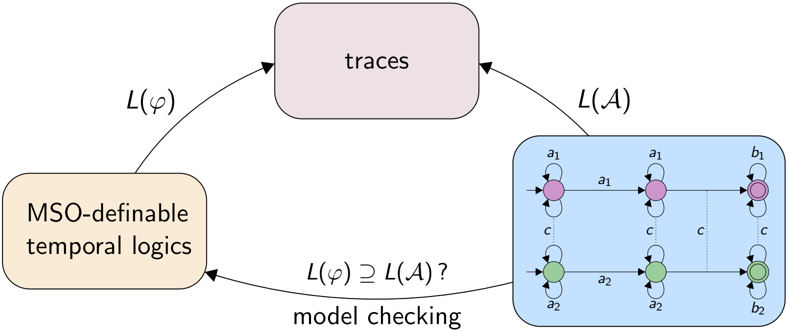

Theorem (Gastin-Kuske '03)

Model checking for any MSO-definable temporal logic is in PSPACE.

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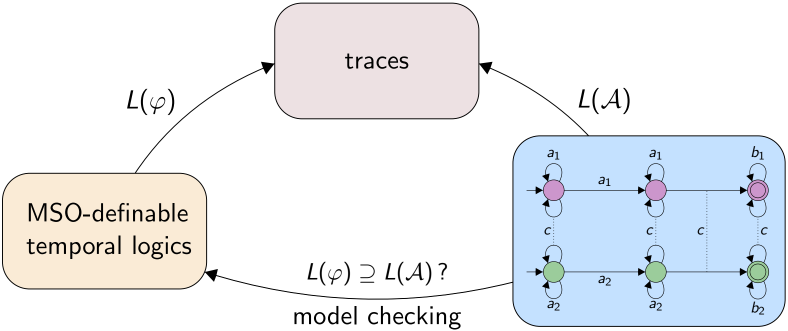


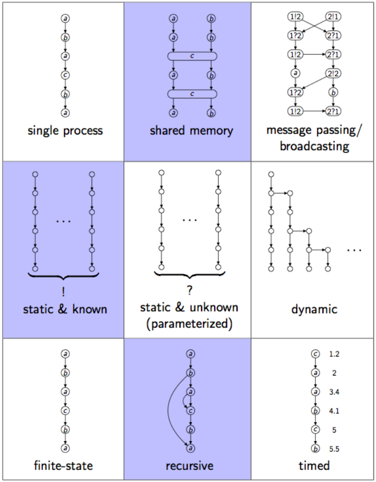
Theorem (Gastin-Kuske '03)

Model checking for any MSO-definable temporal logic is in PSPACE.

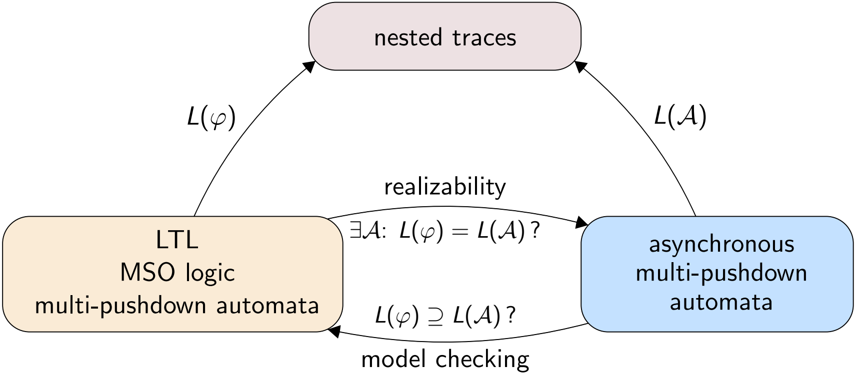
Proof.

Precompile MSO modalities into finite automata. Inductively build finite automaton equivalent to the input formula. □

5. Recursive Shared-Memory Systems



Recursive Shared-Memory Systems



Asynchronous Multi-Pushdown Automata

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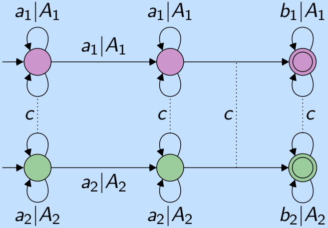
Asynchronous MPA

Stack 1 Stack 2



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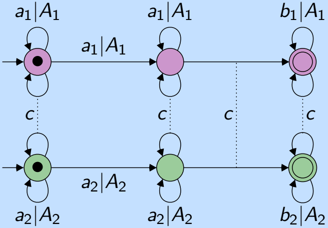

Nested Trace

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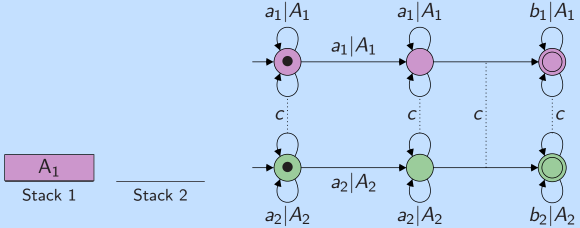

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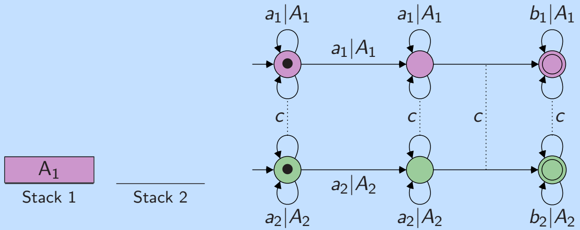

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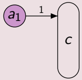

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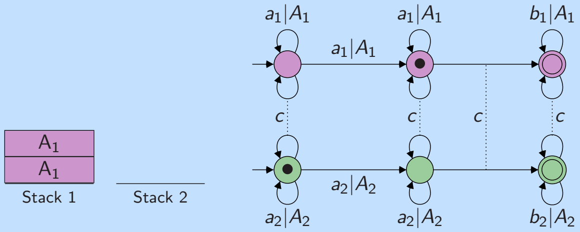

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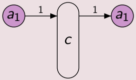

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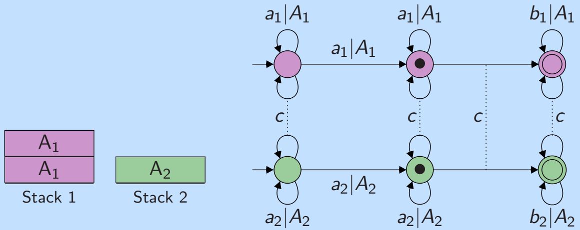

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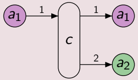

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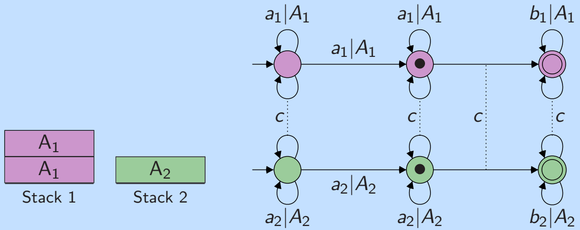

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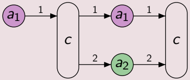

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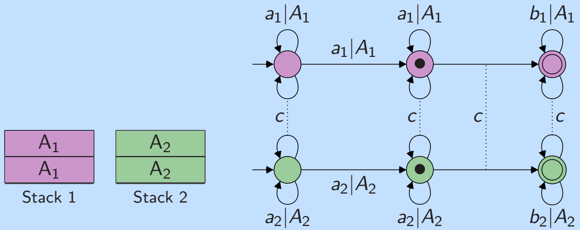

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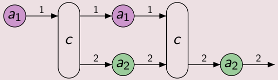

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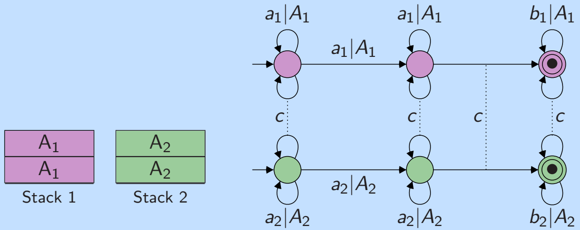

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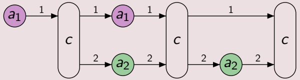

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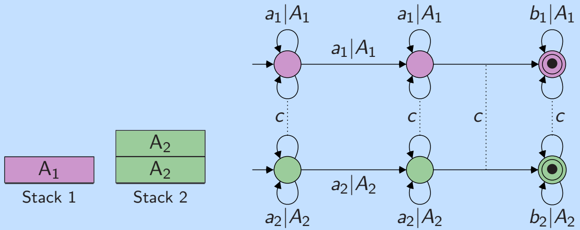

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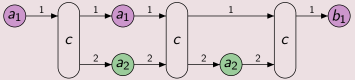

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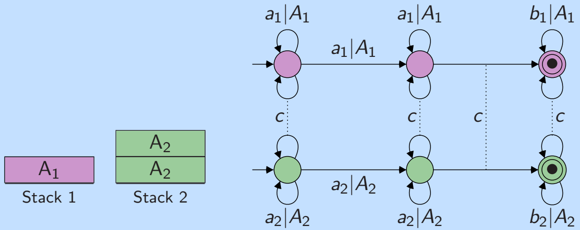

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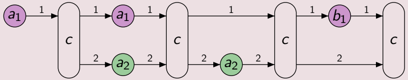

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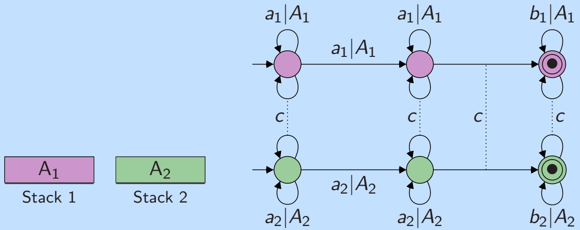

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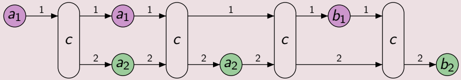

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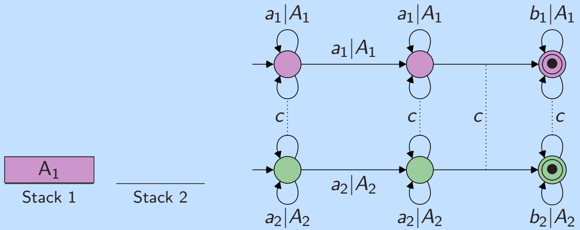

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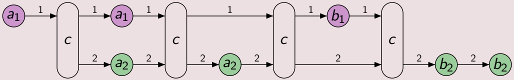

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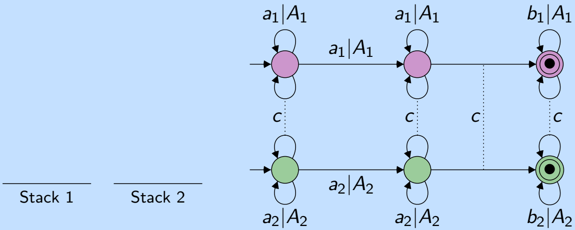

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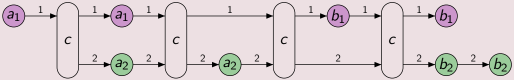

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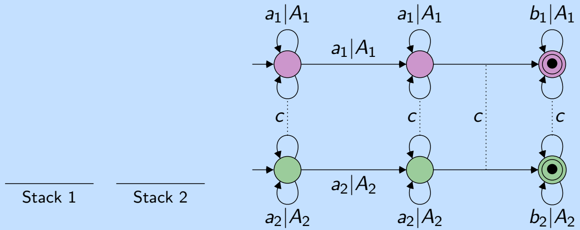

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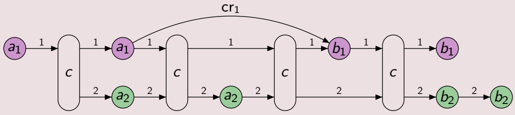

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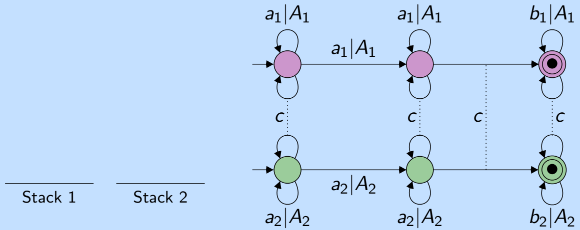

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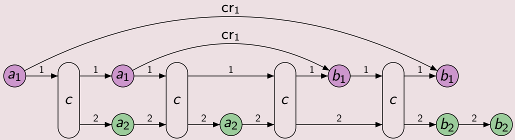

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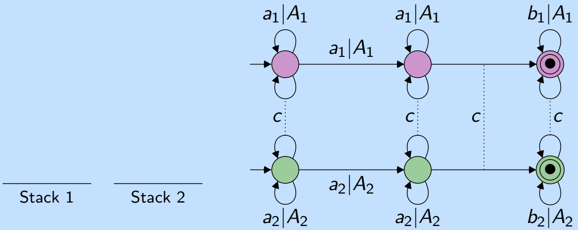

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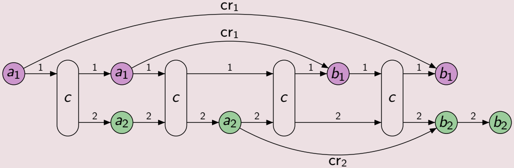

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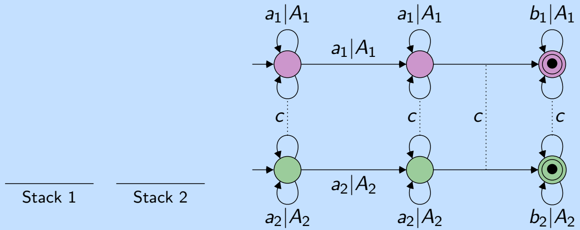

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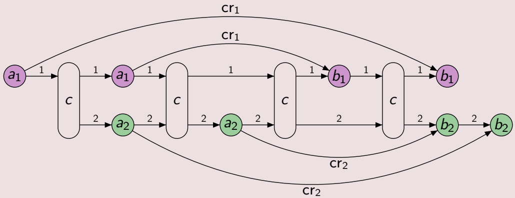

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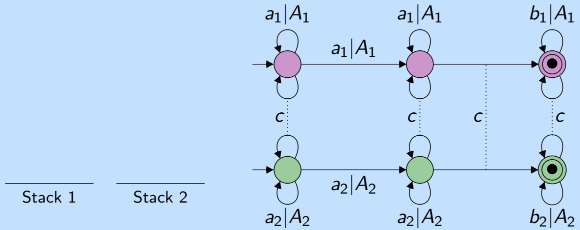

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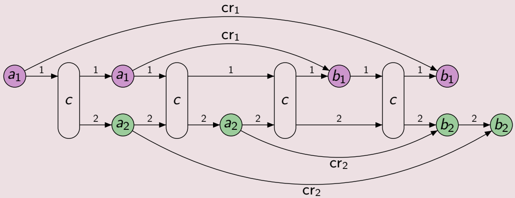
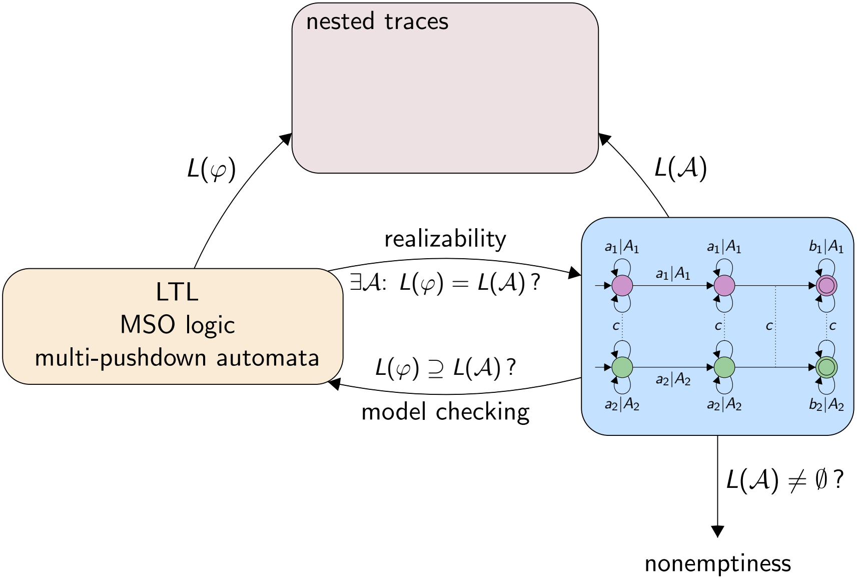


Recursive Shared-Memory Systems



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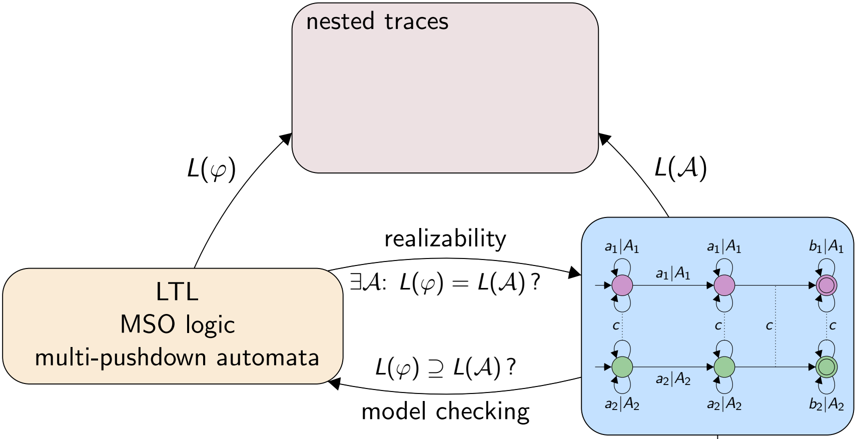

Theorem

Emptiness for (asynchronous) MPA is undecidable.

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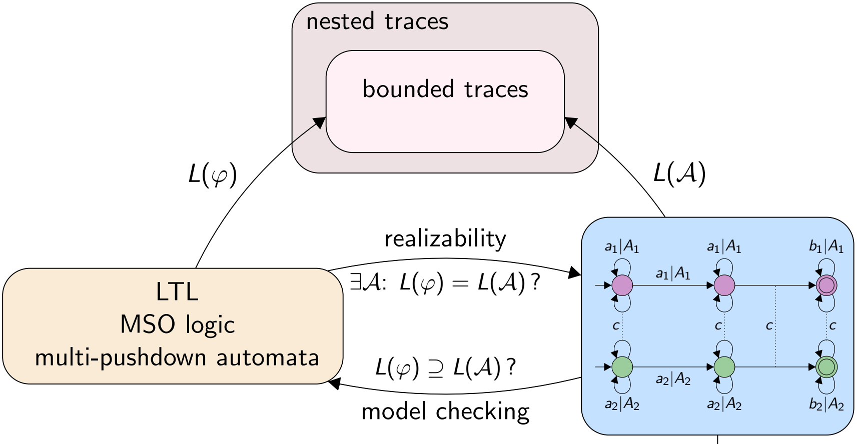

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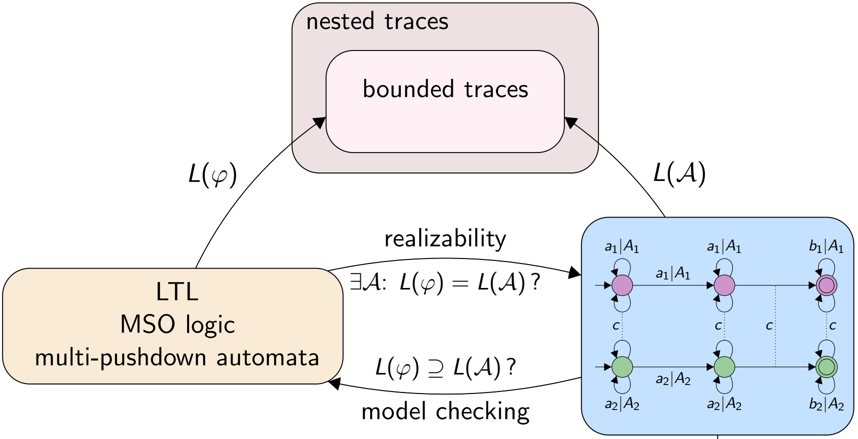

Theorem

Bounded nonemptiness, satisfiability, model checking, and realizability are decidable.

Nested Traces and Their Linearizations

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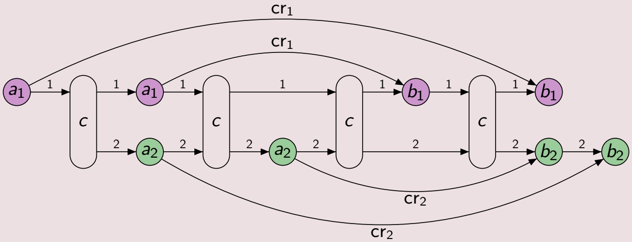

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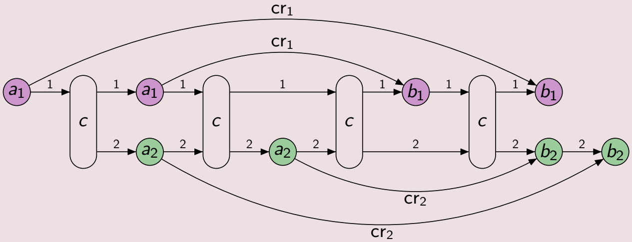
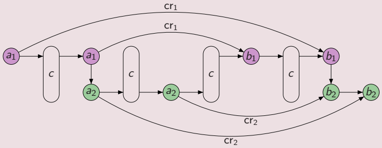


Linearizations $\quad w \in Lin(t) \quad \rightsquigarrow \quad trace(w) = t$



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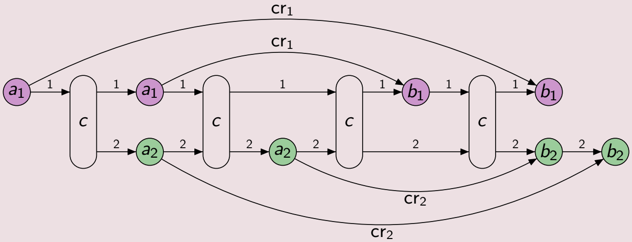

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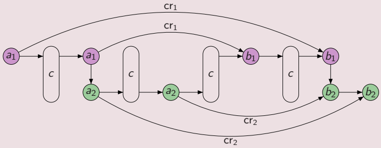
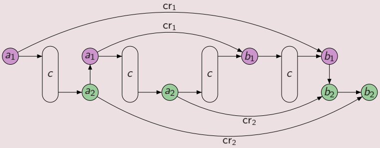


$\sim_0$



Bounded Nested Words

Definition

- In a context, only one process modifies its stack.

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A nested word is

- k-scope bounded if each call-return lies within k contexts.

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- In a context, only one process modifies its stack.
- In a phase, only one process pops from its stack.

A nested word is

- *k*-scope bounded if each call-return lies within k contexts.
- ordered if a pop is performed only on the first nonempty stack.

Bounded Nested Words

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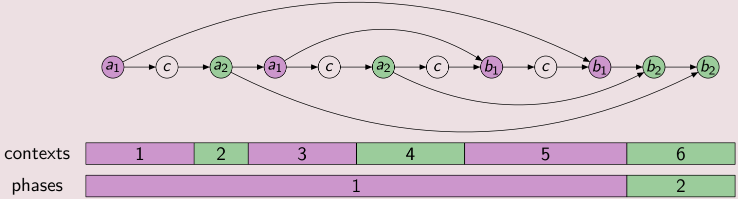
- In a context, only one process modifies its stack.
- In a phase, only one process pops from its stack.

A nested word is

- *k*-scope bounded if each call-return lies within k contexts.
- ordered if a pop is performed only on the first nonempty stack.

Bounded Nested Words



6-context bounded / 2-phase bounded / 5-scope bounded / ordered

Bounded Nested Traces

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A nested trace if k-context bounded / k-phase bounded / k-scope bounded / ordered if at least one linearization is so.

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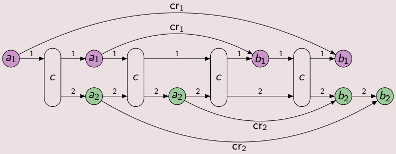
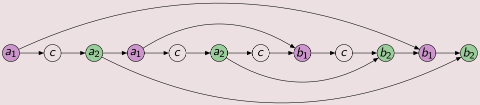
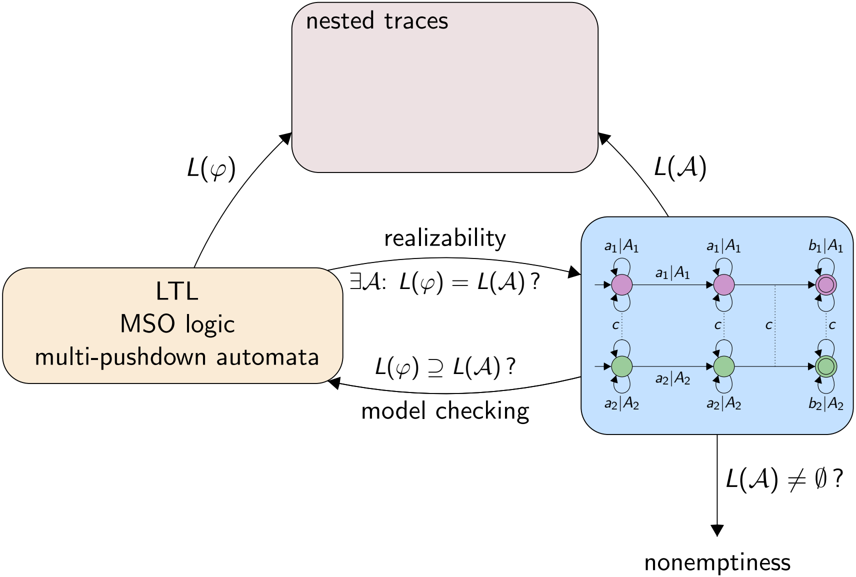


not 2-phase bounded



Recursive Shared-Memory Systems



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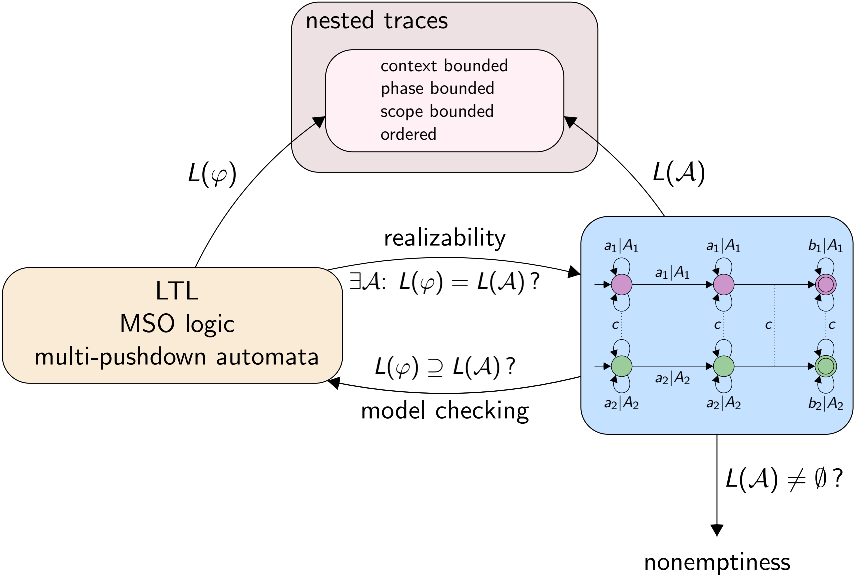

Recursive Shared-Memory Systems

nested traces

context bounded

Theorem

Bounded nonemptiness for sequential MPA is

context	NP-complete	[Qadeer-Rehof '05]
scope	PSPACE-complete	[La Torre-Napoli '11]
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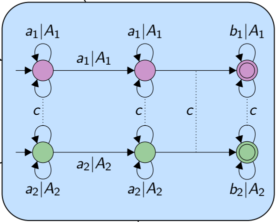
$L(\mathcal{A})$

$\exists\mathcal{A}$: $L(\varphi) = L(\mathcal{A})$?

LTL
MSO logic
multi-pushdown automata

$L(\varphi) \supseteq L(\mathcal{A})$?

model checking



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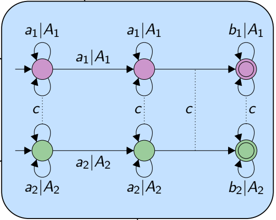
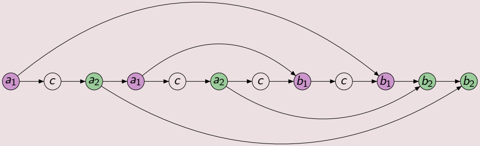


Proof for phases: binary-tree encoding



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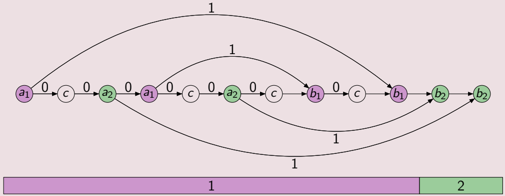

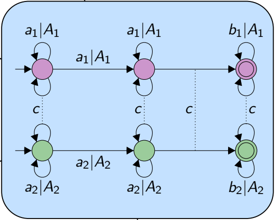

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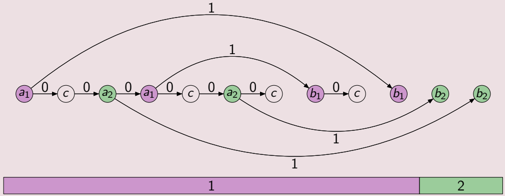

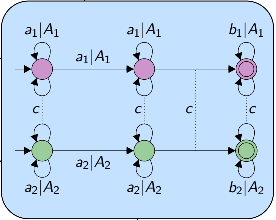

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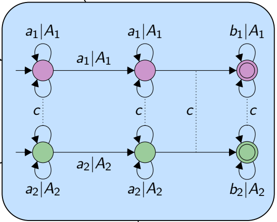

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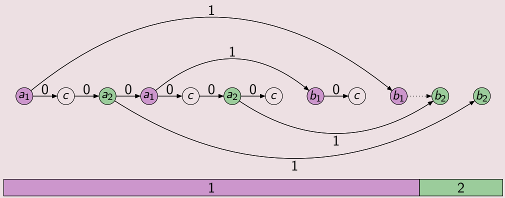

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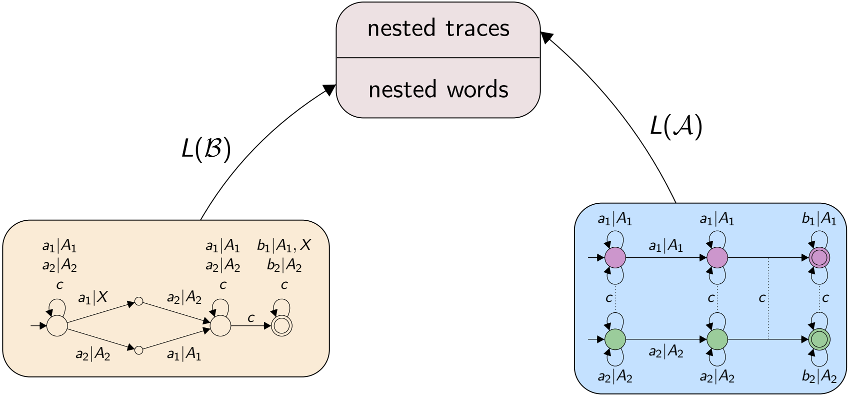

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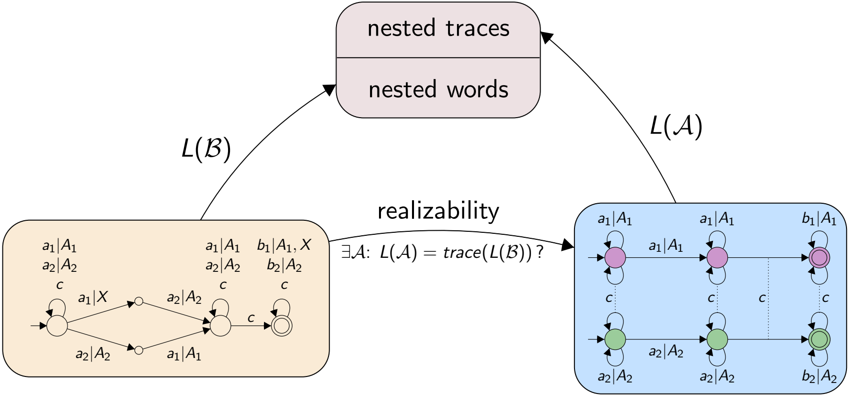

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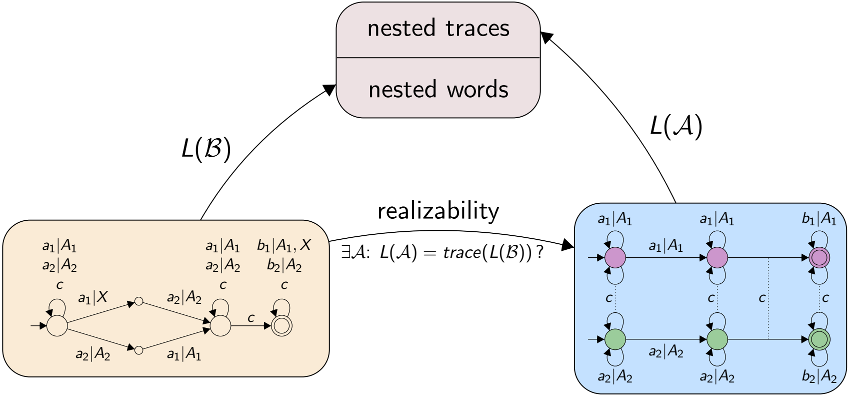
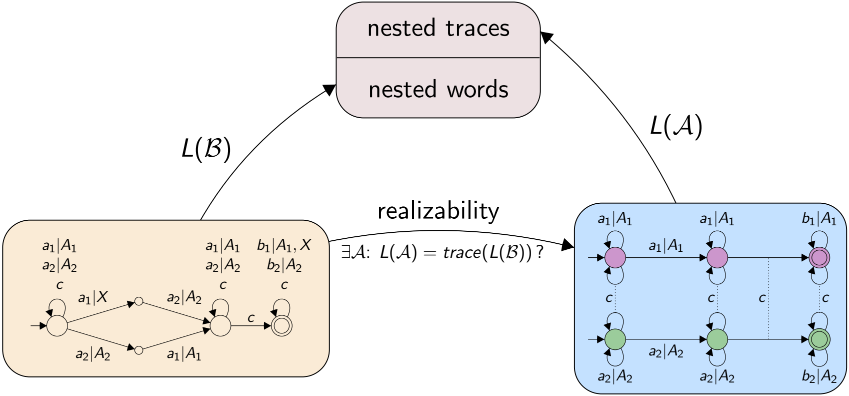


Theorem (B.-Grindei-Habermehl '09)

Let L be a $\sim$-closed language recognized by some sequential MPA. There is an asynchronous MPA $\mathcal{A}$ such that $L(\mathcal{A}) = trace(L)$.

Recursive Shared-Memory Systems



Theorem

It is undecidable if the language of a sequential MPA is $\sim$-closed.

Representations

Let $\theta \in \{k\text{-context}, k\text{-scope}, k\text{-phase}, \text{ordered} \mid k \in \mathbb{N}\}$.

Definition

A set L of θ-nested words is a θ-representation if, for all θ-nested words w, w' with $w \sim_0 w'$, we have $w \in L$ iff $w' \in L$.

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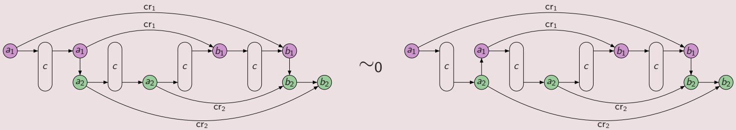

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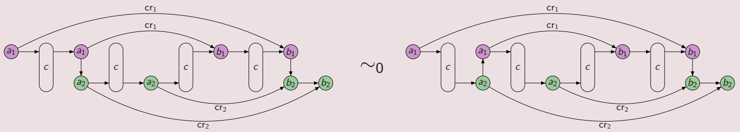

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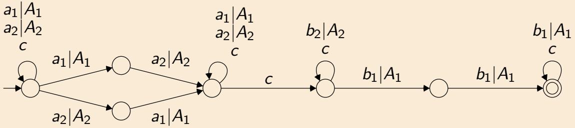

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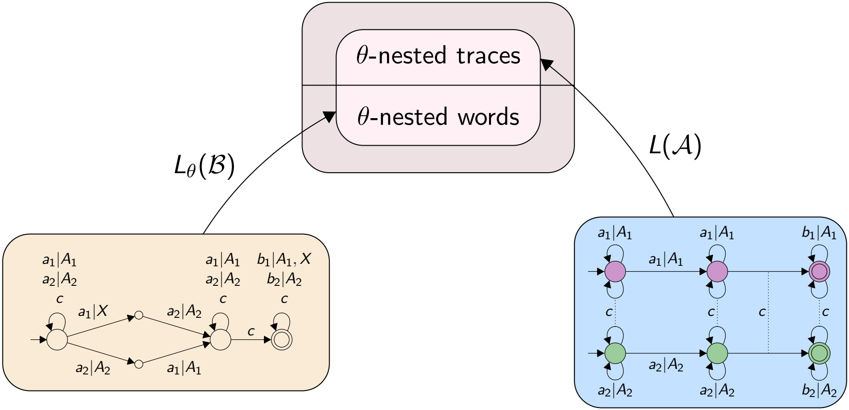

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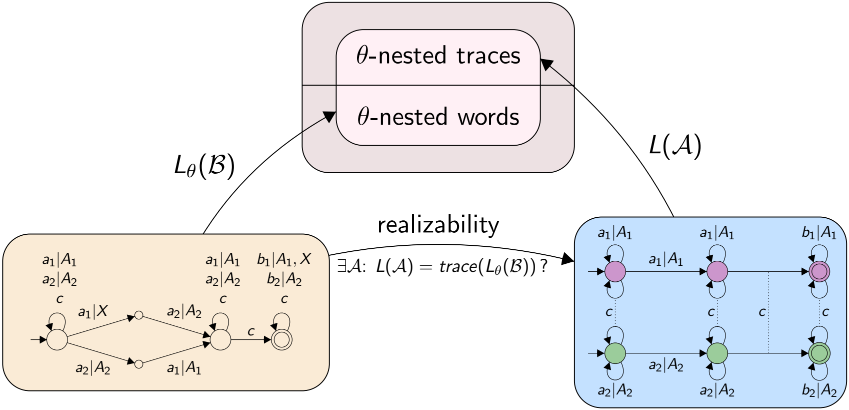

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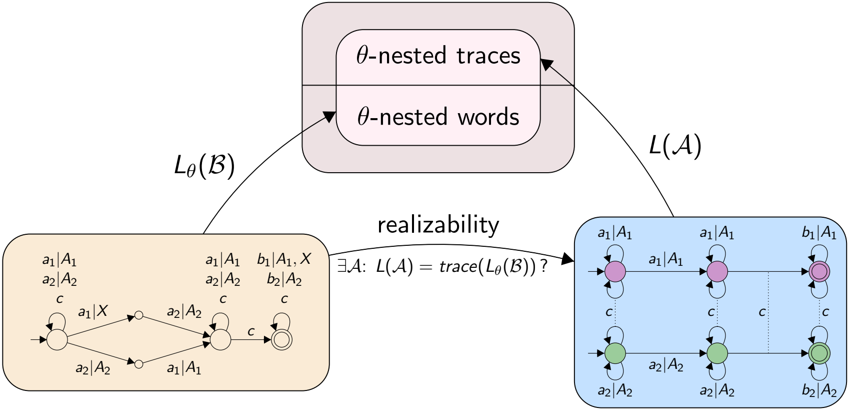
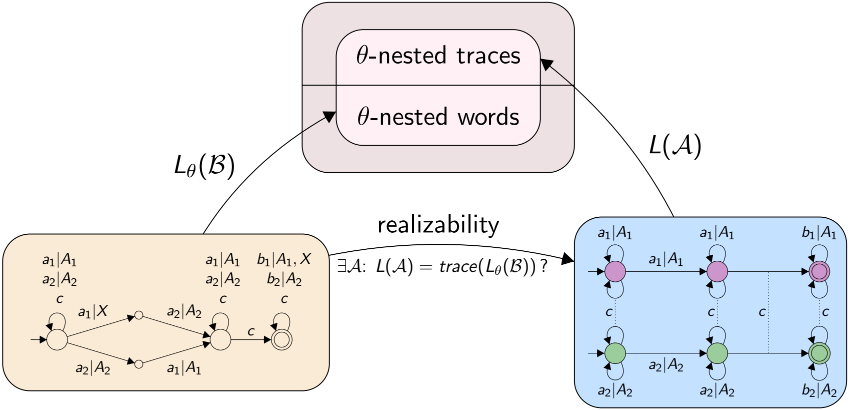


Theorem (B.-Grindei-Habermehl '09)

Let $\mathcal{B}$ be some sequential MPA such that $L_\theta(\mathcal{B})$ is a θ-representation. There is an asynchronous MPA $\mathcal{A}$ such that $L(\mathcal{A}) = trace(L_\theta(\mathcal{B}))$.

Recursive Shared-Memory Systems



Theorem

For a sequential MPA $\mathcal{B}$ it is decidable if $L_\theta(\mathcal{B})$ is a θ-representation (in elementary time).

Monadic Second-Order Logic

Monadic Second-Order Logic (MSO)

- $x \rightarrow_p y$ — x and y are successive events on process $p \in Proc$
- $x \curvearrowright_p y$ — x and y form a call-return pair of process $p \in Proc$
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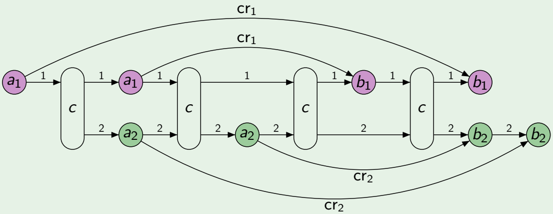

$$\models \; \exists x \exists y \exists z \, (x \curvearrowright_1 y \;\wedge\; a_2(z) \;\wedge\; x \leq z \leq y)$$

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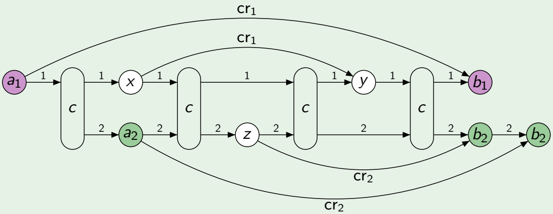

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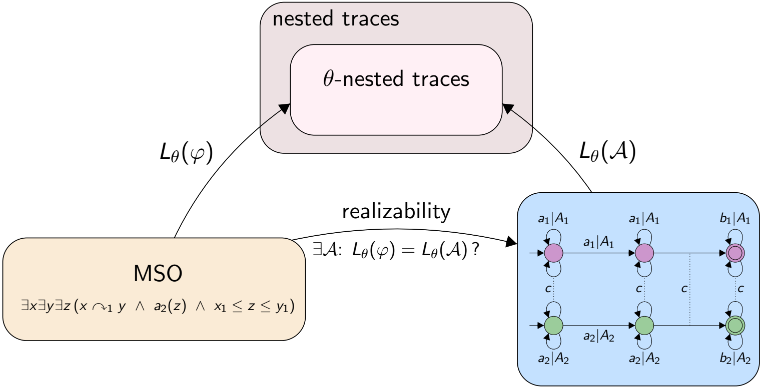

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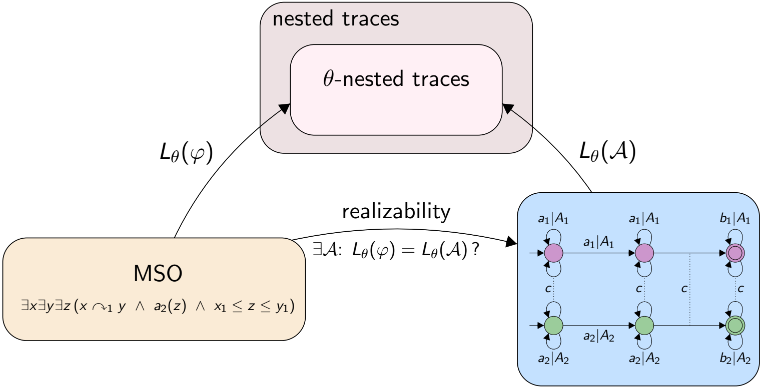

Theorem (La Torre-Madhusudan-Parlato '07-'13)

MSO logic and asynchronous MPA are expressively equivalent wrt. θ-nested traces.

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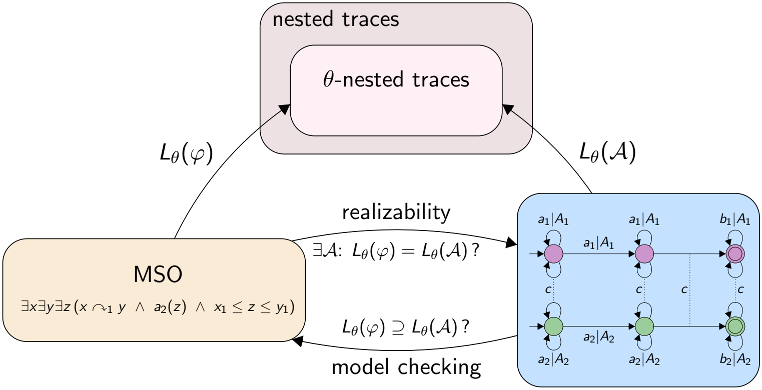

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⇨ MSO model checking is decidable.

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There are lots of (local) temporal logics for nested words/traces!

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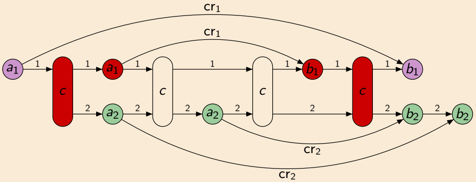
Abstract Until $\varphi \, \mathsf{U}^{\mathsf{a}}_{p} \, \psi$

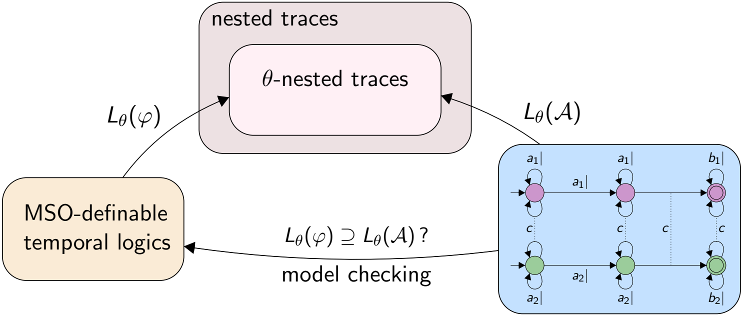


$$\mathrm{MSO}^{\mathsf{U}^{\mathsf{a}}_{p}}(x, X_1, X_2) =$$

$$\exists Y \exists x' \Big(x' \in X_2 \wedge Y \subseteq X_1 \wedge \forall z (z \in Y \vee z = x') \to \big(z = x \vee \exists y \, (y \in Y \wedge \varphi_p(y, z))\big)\Big)$$

where $\varphi_p(y, z) = y \curvearrowright_p z \vee (\neg \exists z' \, y \curvearrowright_p z' \wedge \neg \exists y' \, (y' \curvearrowright_p z \wedge y \to_p z))$.

Model Checking (θ = "k-phase bounded")



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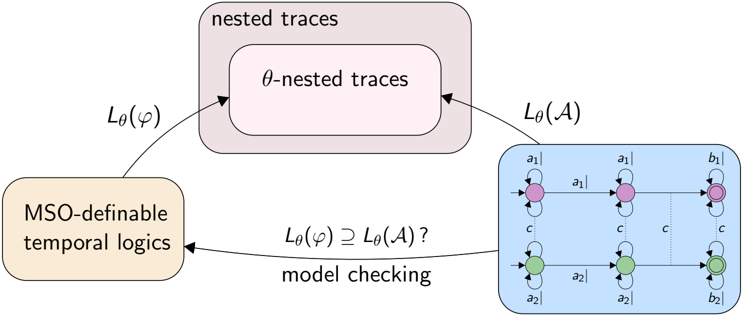

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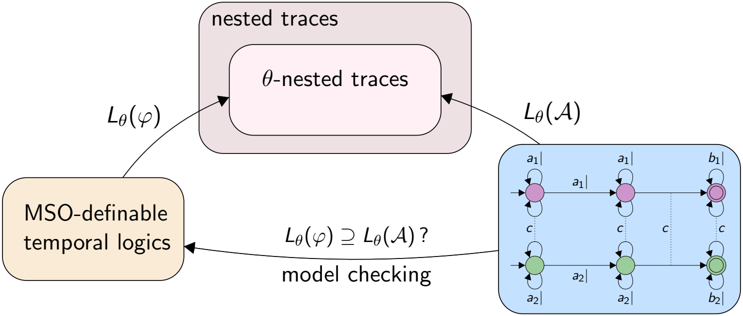

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Theorem (B.-Kuske-Mennicke '13)

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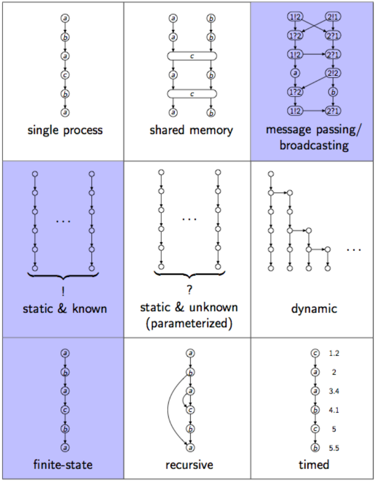
6. Message-Passing Systems



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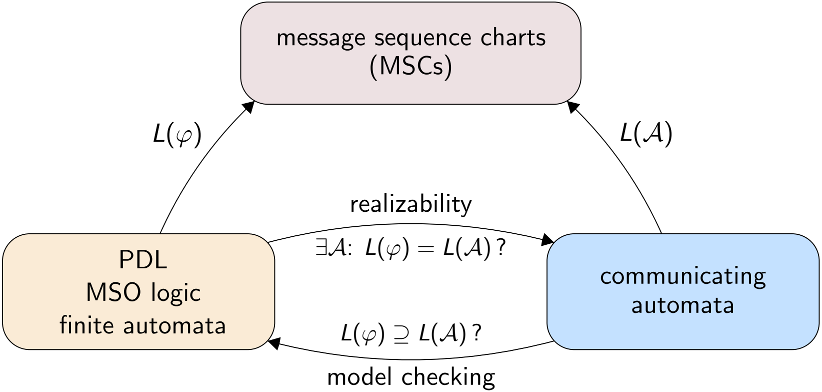

Communicating Automata and MSCs

$Proc = \{1, 2\}$

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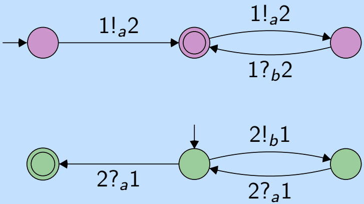

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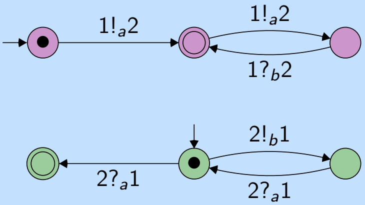

Message Sequence Chart (MSC)

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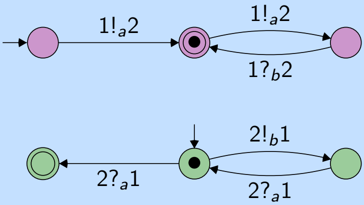

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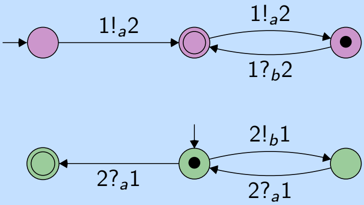

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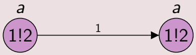

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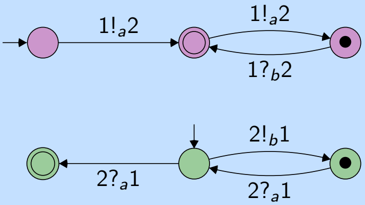

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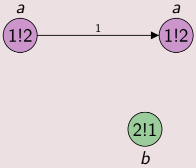

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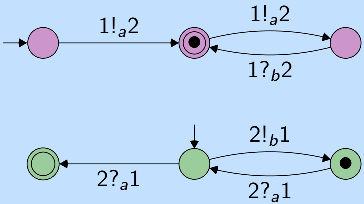

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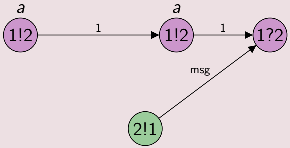

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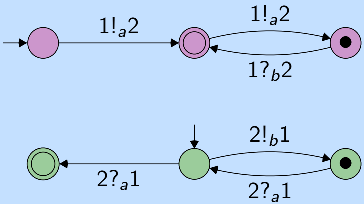

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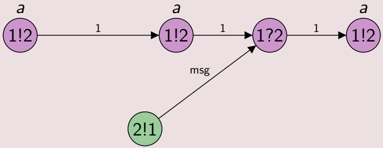

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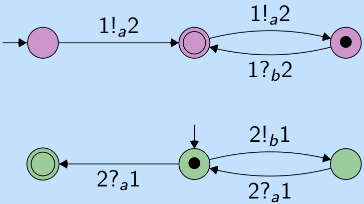
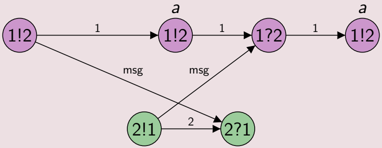
Communicating Automaton



Message Sequence Chart (MSC)



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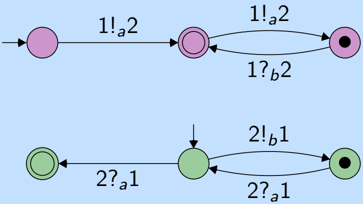

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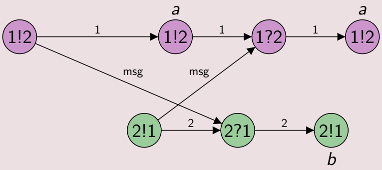

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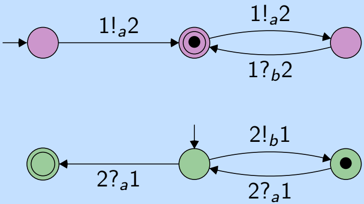

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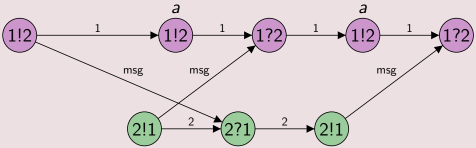

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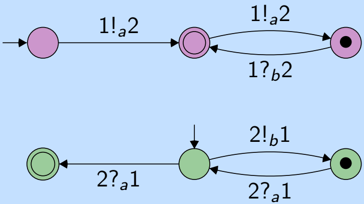

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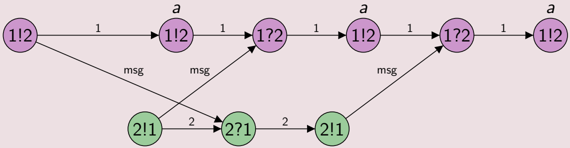

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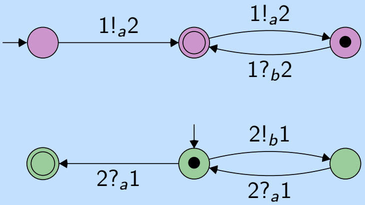

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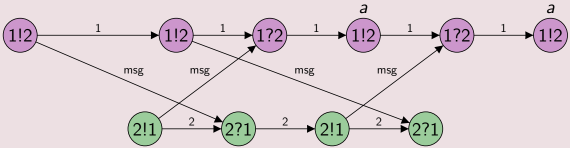

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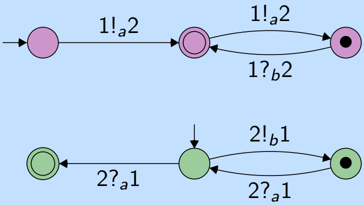

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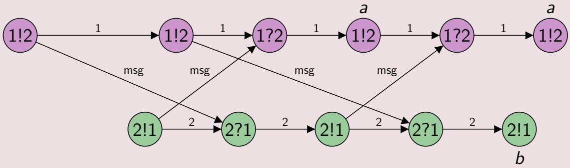

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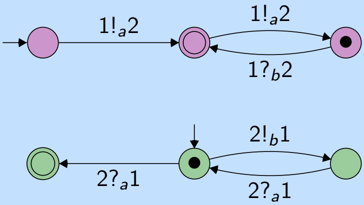

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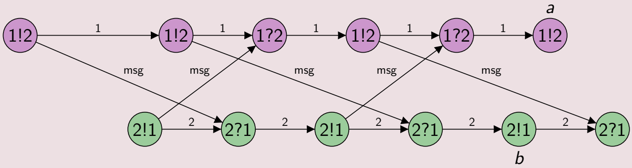

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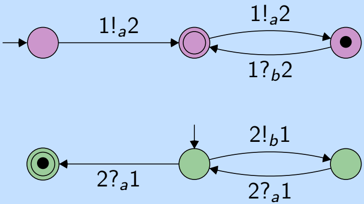

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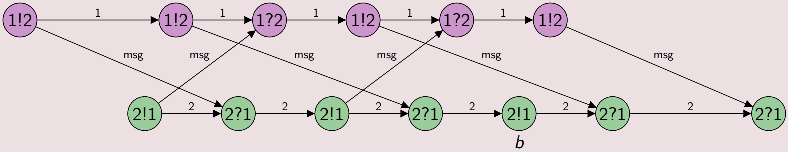

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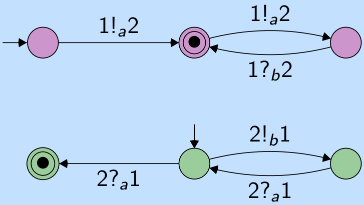

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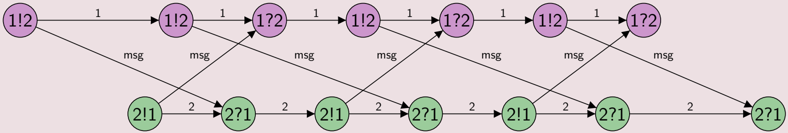

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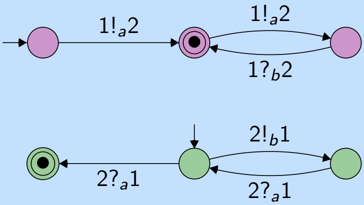
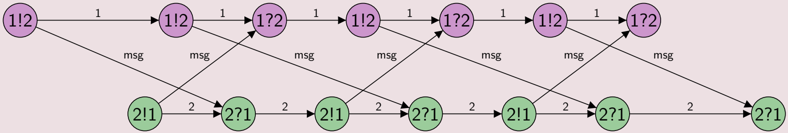
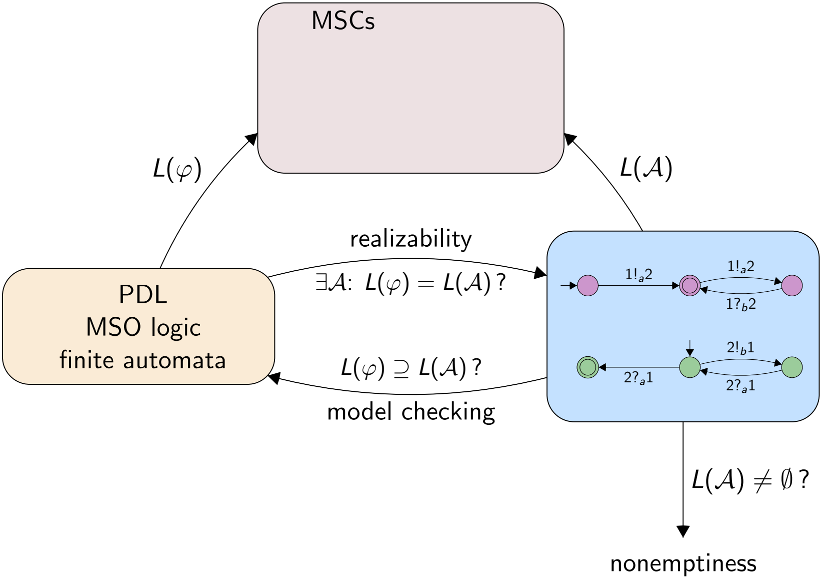


Message Sequence Chart (MSC) $\quad M = (E, \to_1, \to_2, \xrightarrow{\text{msg}}, \lambda)$



Message-Passing Systems



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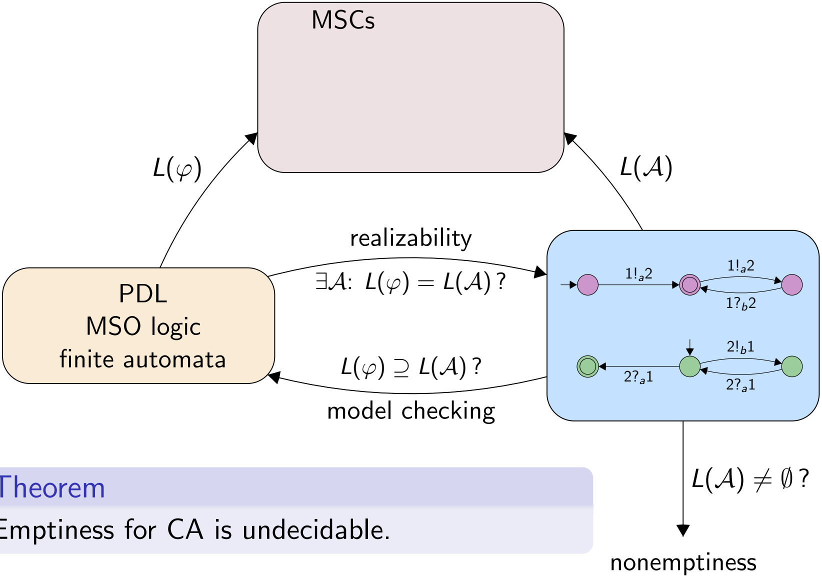
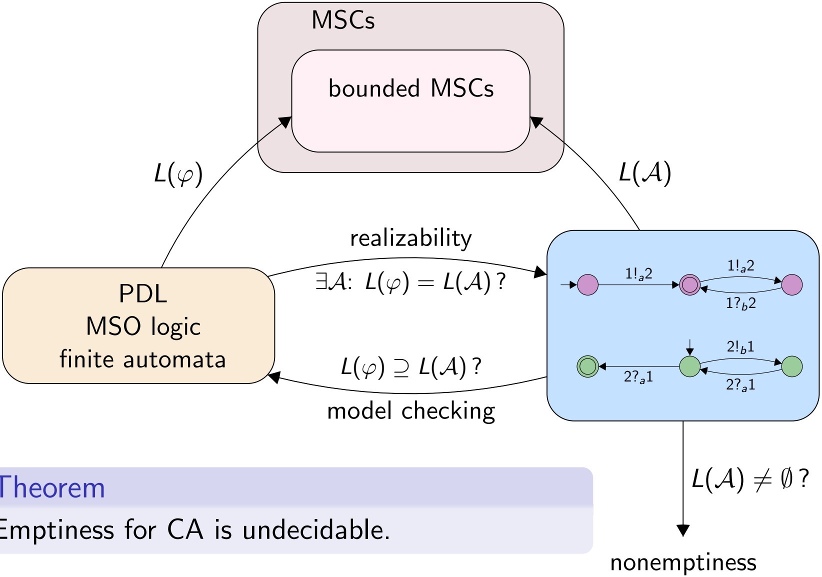


Theorem

Emptiness for CA is undecidable.

Message-Passing Systems



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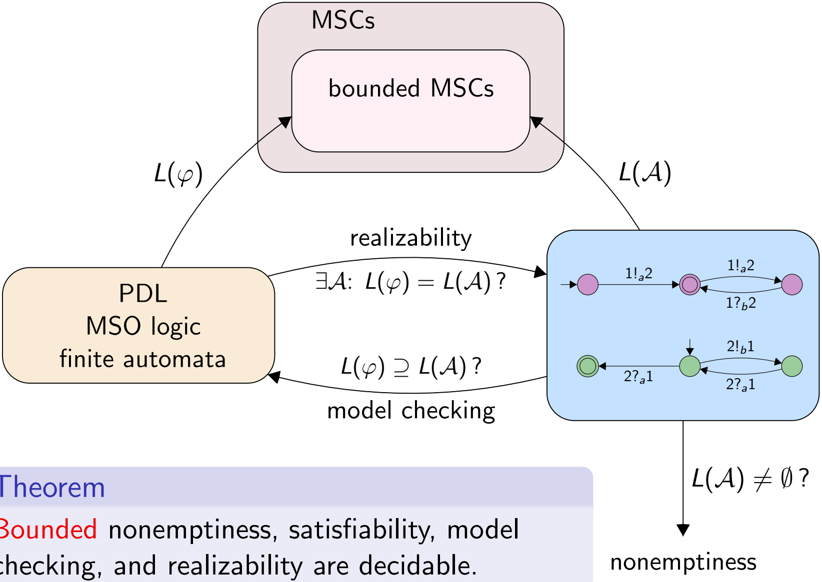
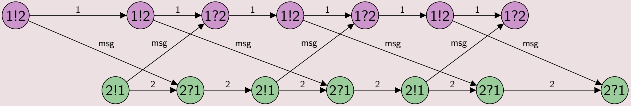


Theorem

Bounded nonemptiness, satisfiability, model checking, and realizability are decidable.

Channel-Bounded MSCs

MSC M



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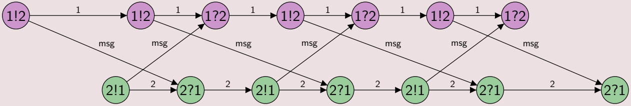
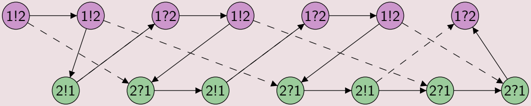


3-bounded linearization $w \in Lin(M) \subseteq \Sigma^*$ $\rightsquigarrow$ $msc(w) = M$



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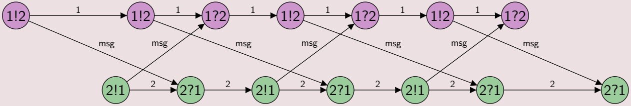
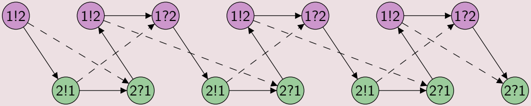


1-bounded linearization $w \in Lin(M) \subseteq \Sigma^*$ $\rightsquigarrow$ $msc(w) = M$



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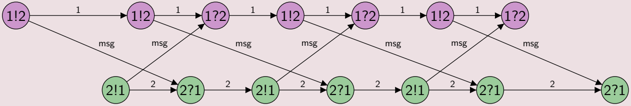

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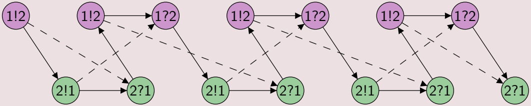

Definition

Let $B \in \mathbb{N}$. An MSC is

- $\exists B$-bounded if some linearization is B-bounded linearization.
- $\forall B$-bounded if every linearization is B-bounded.

Representations

Definition

A set $L \subseteq \Sigma^*$ (of well-formed words) is a

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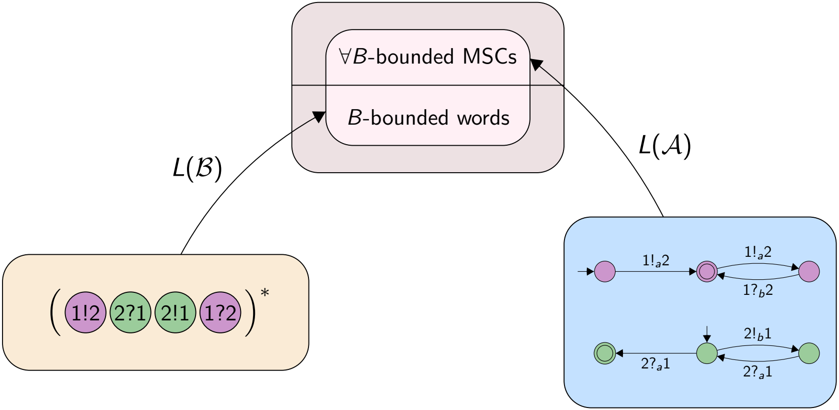
$\left(1!2 \; 2?1 \right)^*$ is an $\exists 1$-representation, but no $\forall$-representation.

$\left(1!2 \; 2?1 \quad 3!4 \; 4?3 \right)^*$ is not an $\exists B$-representation, for any B.

Message-Passing Systems



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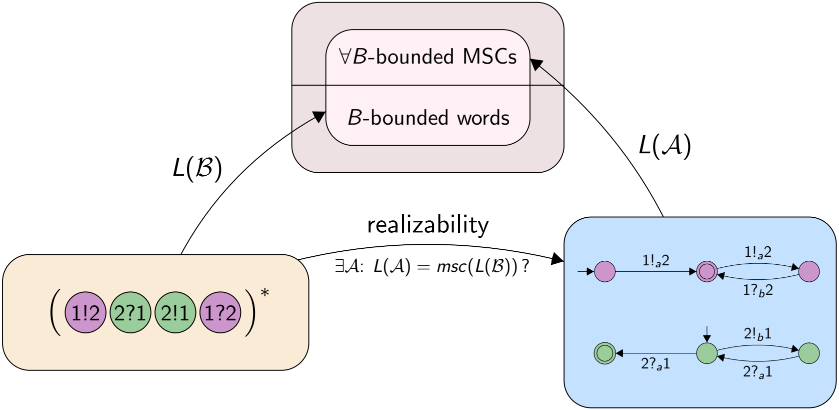

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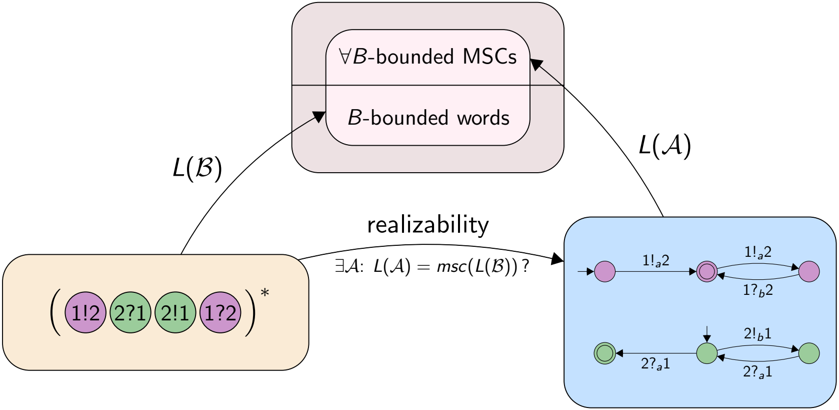
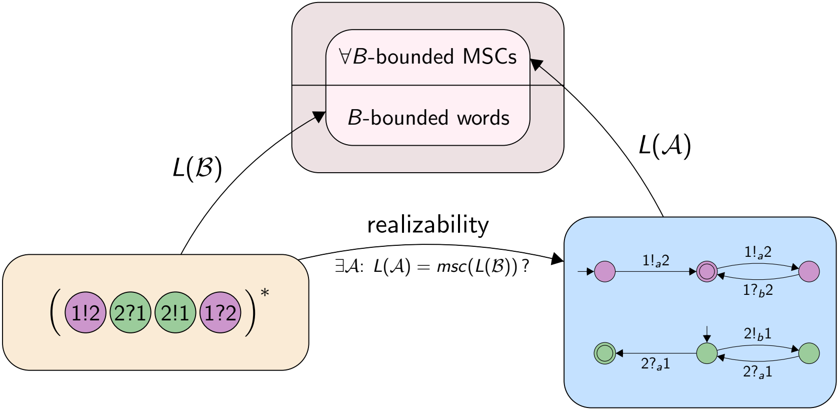


Theorem (Henriksen et al. '00; Kuske '03)

Let $\mathcal{B}$ be some finite automaton such that $L(\mathcal{B})$ is a $\forall$-representation.
There is a (deterministic) CA $\mathcal{A}$ such that $L(\mathcal{A}) = \mathit{msc}(L(\mathcal{B}))$.

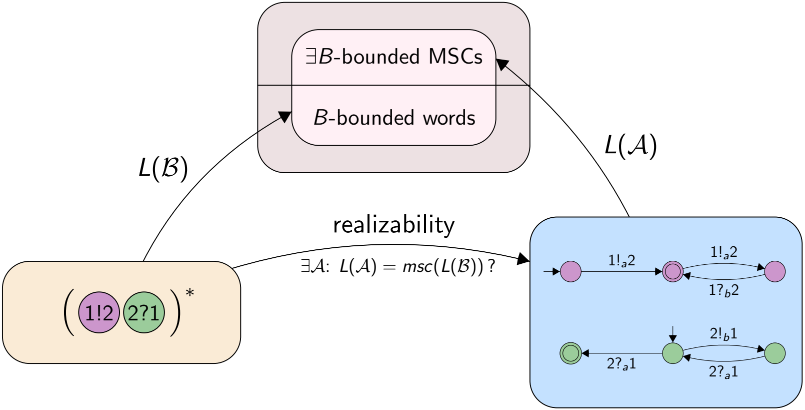
Message-Passing Systems



Theorem (Henriksen et al. '00)

For a finite automaton $\mathcal{B}$ it is decidable if $L(\mathcal{B})$ is a $\forall$-representation.

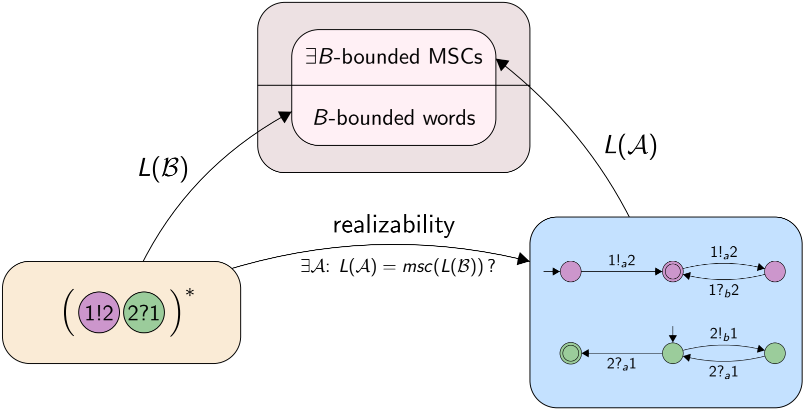
Message-Passing Systems



Theorem (Genest-Kuske-Muscholl '06)

Let $\mathcal{B}$ be some finite automaton such that $L(\mathcal{B})$ is a $\exists B$-representation. There is a CA $\mathcal{A}$ such that $L(\mathcal{A}) = msc(L(\mathcal{B}))$.

Message-Passing Systems



Theorem

For a finite automaton $\mathcal{B}$ it is decidable if $L(\mathcal{B})$ is an $\exists B$-representation.

Monadic Second-Order Logic

Monadic Second-Order Logic (MSO)

- $x \rightarrow_p y$ — x and y are successive events on process $p \in Proc$
- $x \stackrel{\text{msg}}{\rightarrow} y$ — x and y form a message
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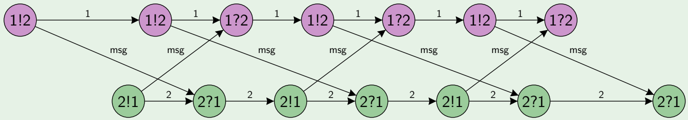
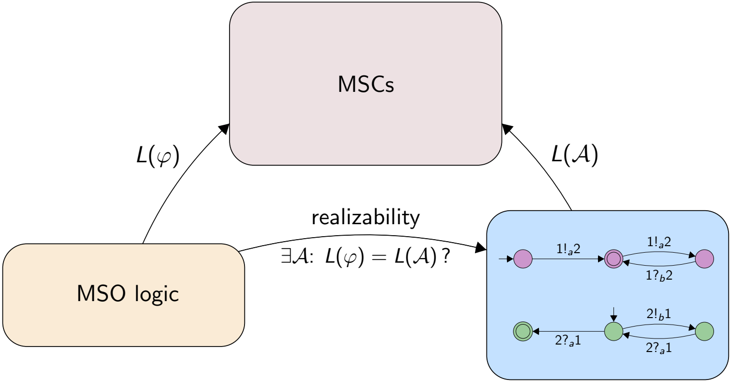
Example



$$\models \; \exists x, y, x', y' \, (x \xrightarrow{\text{msg}} y \wedge x' \xrightarrow{\text{msg}} y' \wedge x \rightarrow_1^* y' \wedge x' \rightarrow_2^* y)$$

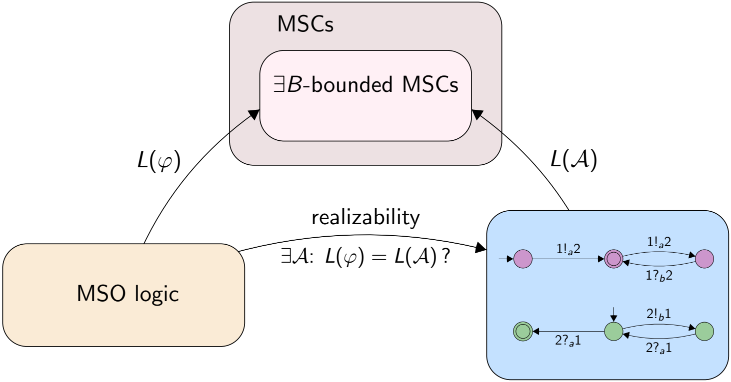
Message-Passing Systems



Theorem (B.-Leucker '04)

EMSO logic ($\exists X_1 \ldots X_n \varphi$ with φ first-order) and communicating automata are expressively equivalent. MSO logic is strictly more expressive.

Message-Passing Systems

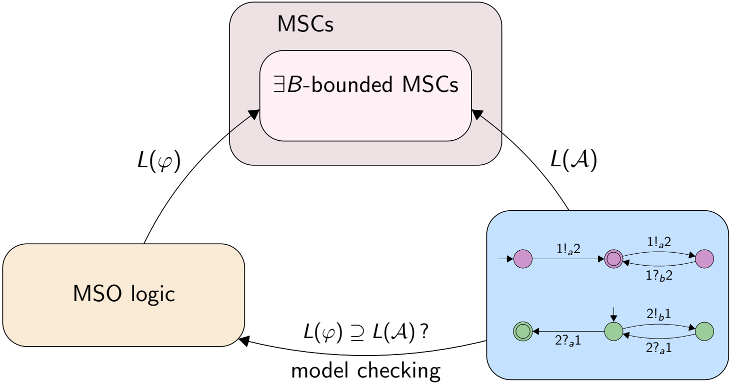


Theorem (Genest-Kuske-Muscholl '04)

Let L be a set of $\exists B$-bounded MSCs. The following are equivalent:

- There is an MSO sentence φ such that $L = L(\varphi)$.
- There is a CA $\mathcal{A}$ such that $L = L(\mathcal{A})$.

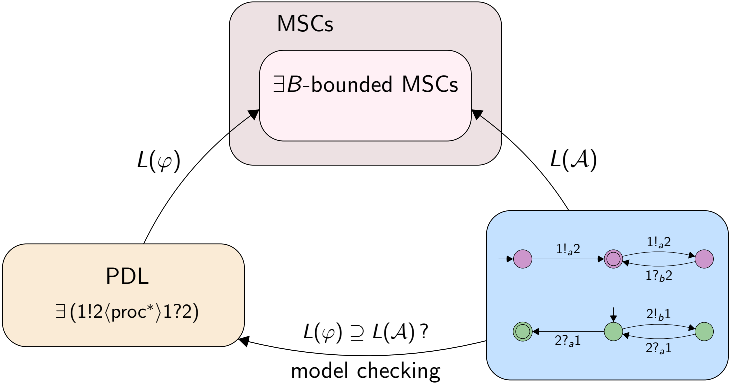
Message-Passing Systems



Theorem (Genest-Kuske-Muscholl '04)

Given a CA $\mathcal{A}$ and an MSO sentence φ, it is decidable if all $\exists B$-bounded MSCs from $L(\mathcal{A})$ satisfy φ.

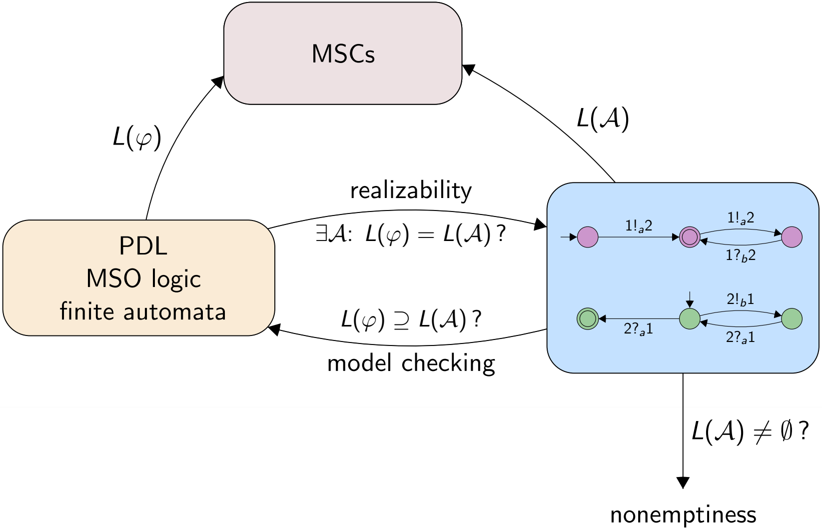
Message-Passing Systems



Theorem (B., Kuske, Meinecke 2007; Mennicke 2012)

Given a CA $\mathcal{A}$ and a PDL formula φ, it is decidable in PSPACE if all $\exists B$-bounded MSCs from $L(\mathcal{A})$ satisfy φ.

Message-Passing Systems



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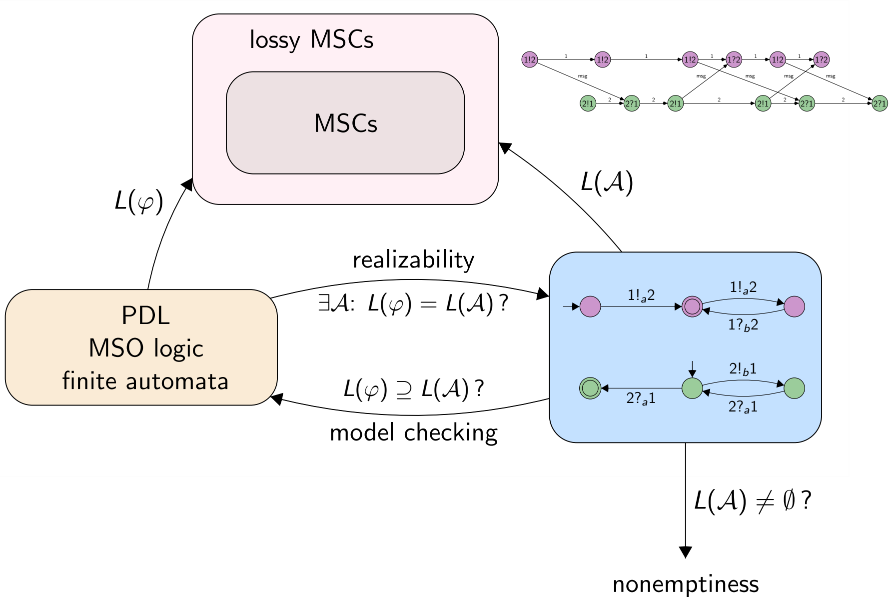

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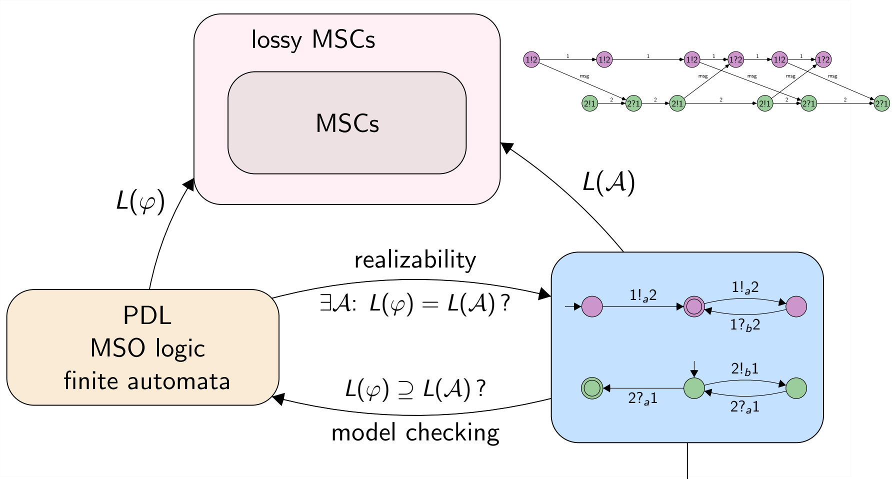
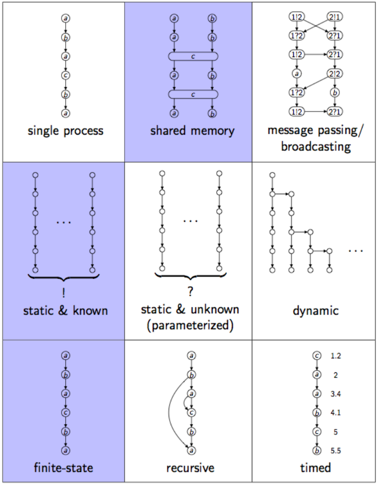


Theorem (Finkel '87, Abdulla-Jonsson '96)

Emptiness for lossy CA is decidable.

7. Conclusion and Perspectives

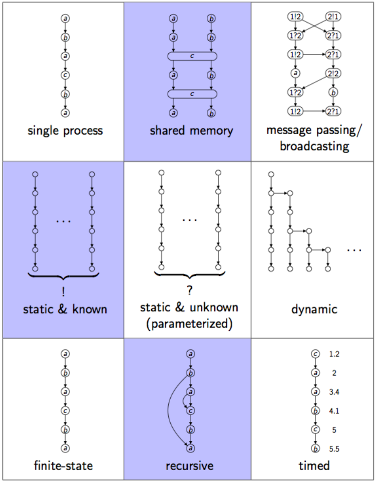
Conclusion: Finite-State Shared-Memory Systems



Realizability ✔

Model Checking ✔

Conclusion: Recursive Shared-Memory Systems



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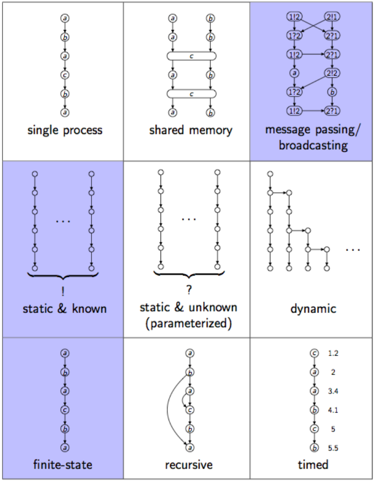
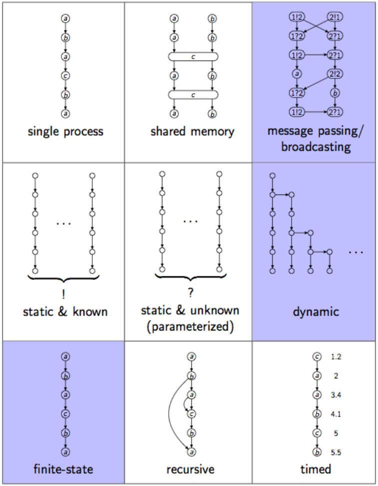
Conclusion: Message-Passing Systems



Realizability ✔

Model Checking ✔

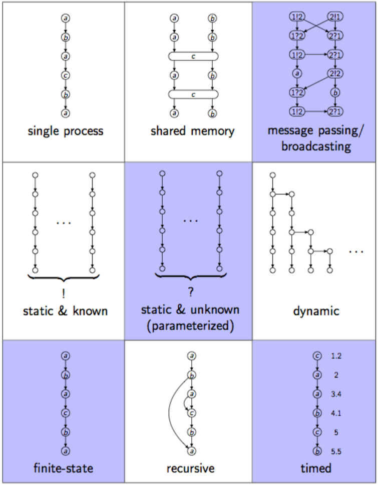
Perspectives: Dynamic Message-Passing Systems



Realizability ✔ ✗

Model Checking ✔ ✗

Perspectives: Parameterized Systems



Realizability ✗

Model Checking ✓ ✗

Reachability ✓

Thank You!