

# Main theorems

**Theorem 3.3.5** There exists an algorithm that solves the following decision problem:

- input:** 1. an alphabet  $\Sigma$ ,  
 2. a basis algorithm of an effective and monotone  $\Sigma$ -ACM  $\mathcal{A}'$ ,  
 3. the set of final states  $F'$  of  $\mathcal{A}'$ ,  
 4. a finite basis of  $(Q_c, \sqsubseteq_c)$ , and an algorithm to decide  $\sqsubseteq_c$  for  $c \in \Sigma$ .

**output:** Is  $L(\mathcal{A}')$  empty?

**Corollary 3.3.6** Let  $\mathcal{A}$  be a monotone and effective  $\Sigma$ -ACM. Then the set  $L(\mathcal{A})$  is recursive.

**Theorem 4.1.7** Let  $\Sigma$  be an alphabet with at least two letters. Then there is no algorithm that on input of a  $\Sigma$ -ACA  $\mathcal{A}$  decides whether it accepts all  $\Sigma$ -dags, i.e. whether  $L(\mathcal{A}) = \mathbb{D}$ .

**Corollary 4.1.8** Let  $\Sigma$  be an alphabet with at least two letters. Then the equivalence of  $\Sigma$ -ACAs, i.e. the question whether  $L(\mathcal{A}_1) = L(\mathcal{A}_2)$ , is undecidable.

**Theorem 4.1.10** Let  $\Sigma$  be an alphabet with at least two letters. Then there is no algorithm that on input of a  $\Sigma$ -ACA  $\mathcal{A}$  decides any of the following questions:

1. Is  $\mathbb{D} \setminus L(\mathcal{A})$  recognizable?
2. Is  $\mathcal{A}$  equivalent with some deterministic  $\Sigma$ -ACA?

**Theorem 5.1.1** Let  $\mathcal{A}$  be a possibly nondeterministic  $\Sigma$ -ACA. There exists a monadic sentence  $\varphi$  over  $\Sigma$  such that  $L(\mathcal{A}) = \{t \in \mathbb{D} \mid t \models \varphi\}$ .

**Theorem 5.2.10** Let  $\varphi$  be a monadic sentence and let  $k \in \mathbb{N}$ . Then there exists a  $\Sigma$ -ACA  $\mathcal{A}$  such that  $L(\mathcal{A}) = \{t \in \mathbb{D}_k \mid t \models \varphi\}$ .

**Theorem 6.1.5** Let  $L \subseteq \text{SP}(\Sigma)$  be a width-bounded sp-language. Then  $L$  can be accepted by a branching automaton iff it is monadically axiomatizable.

**Corollary 6.1.8** Let  $\mathcal{B}$  be a branching automaton. Then there exists a  $\Sigma$ -ACA  $\mathcal{A}$  such that  $\text{Ha}(L(\mathcal{B})) \cap \mathbb{D} = L(\mathcal{A})$ .

**Corollary 6.1.6** Let  $\mathcal{A}$  be a  $\Sigma$ -ACA. Then there exists a branching automaton  $\overline{\mathcal{B}}$  such that  $\text{Ha}(L(\overline{\mathcal{B}})) = L(\mathcal{A}) \cap \text{Ha}(\text{SP}(\Sigma))$ .

**Theorem 6.2.1** Let  $\mathcal{B}$  be a P-asynchronous automaton over  $\Sigma$ . Then there exists a  $\Sigma$ -ACA  $\mathcal{A}$  with  $\text{Ha}(L(\mathcal{B})) = L(\mathcal{A})$ .

**Theorem 8.2.10** Let  $T$  be a finite set and  $E$  a set of equations of the form  $ab = cd$  with  $a, b, c, d \in T$ . Let  $\sim$  be the least congruence on  $T^*$  containing  $E$ . Then  $M := T^*/\sim$  is a divisibility monoid if and only if (i)-(iii) hold for any  $a, b, c, b', c' \in T$ :

- (i)  $(\downarrow(a \cdot b \cdot c), \leq)$  is a distributive lattice,
- (ii)  $a \cdot b \cdot c = a \cdot b' \cdot c'$  or  $b \cdot c \cdot a = b' \cdot c' \cdot a$  implies  $b \cdot c = b' \cdot c'$ , and
- (iii)  $a \cdot b = a' \cdot b'$ ,  $a \cdot c = a' \cdot c'$  and  $a \neq a'$  imply  $b = c$ .

Furthermore, each divisibility monoid arises this way.

**Theorem 9.1.8** Let  $(M, \cdot, 1)$  be a divisibility monoid with finitely many residuum functions. Let  $X \subseteq T^*$  be recognizable and of finite rink. Then  $\text{nat}(X)$  is recognizable in  $M$ .

**Theorem 9.2.8** Let  $(M, \cdot, 1)$  be a divisibility monoid with finitely many residuum functions. Let  $L \subseteq M$  be c-rational. Then  $L$  is recognizable.

**Theorem 9.3.8** Let  $(M, \cdot, 1, \rho)$  be a labeled divisibility monoid finitely many residuum functions. Let  $L \subseteq M$ . Then the following are equivalent:

- 1.  $L$  is recognizable
- 2.  $L$  is c-rational
- 3.  $L$  is mc-rational.

**Theorem 10.2.8** Let  $(M, \cdot, 1)$  be a divisibility monoid with finitely many residuum functions. Then the following are equivalent

- 1.  $M$  is width-bounded,
- 2.  $M$  is rational, and
- 3. any set  $L \subseteq M$  is rational iff it is recognizable.

**Theorem 11.1.3** Let  $(M, \cdot, 1)$  be a divisibility monoid finitely many residuum functions. Then the monadic theory  $\text{MTh}(\{(\mathbb{J}(\downarrow m), <) \mid m \in M\})$  is decidable.

**Theorem 11.1.4** Let  $(\Sigma, D)$  be a finite dependence alphabet. Then the monadic theory of  $(\mathbb{J}(\mathbb{M}(\Sigma, D)), \leq)$  is decidable iff  $D$  is transitive.

**Theorem 11.1.9** Let  $(M, \cdot, 1)$  be a divisibility monoid with finitely many residuum functions. Then the monadic theory  $\text{MTh}\{(\downarrow m, \leq) \mid m \in M\}$  is decidable iff  $M$  is width-bounded.

**Theorem 11.2.2** Let  $\mathfrak{P}$  be a set of partially ordered sets of uniformly bounded width. Then  $\text{Th}(\mathbb{H}_f(\mathfrak{P}))$  can be reduced to  $\text{Th}(\mathfrak{P})$  in linear time.

**Theorem 11.2.10** Let  $\mathfrak{P}$  be a set of partially ordered sets whose diabolo width is uniformly bounded. Then  $\text{MTh}(\mathbb{H}_f(\mathfrak{P}))$  can be reduced to  $\text{MTh}(\mathfrak{P})$  in linear time.

**Corollary 11.3.1** Let  $\mathfrak{L}$  be a set of finite distributive lattices.

1. The following are equivalent:
  - (i) The monadic theory  $\text{MTh}(\mathfrak{L})$  is decidable.
  - (ii) The monadic chain theory  $\text{MCTh}(\mathfrak{L})$  is decidable.
  - (iii) the monadic theory  $\text{MTh}(\mathbb{J}(\mathfrak{L}))$  is decidable and the width of the elements of  $\mathfrak{L}$  is bounded above.
  - (iv) the monadic chain theory  $\text{MCTh}(\mathbb{J}(\mathfrak{L}))$  is decidable and the width of the elements of  $\mathfrak{L}$  is bounded above.
2. The monadic antichain theory  $\text{MATH}(\mathfrak{L})$  is decidable if and only if the elementary theory  $\text{Th}(\mathbb{J}(\mathfrak{L}))$  is decidable and the width of the elements of  $\mathfrak{L}$  is bounded above.

# Open problems

In the introduction, I explained that the two faces of Mazurkiewicz traces (dependence graphs and free partially commutative monoids) are the bases for their rich theory. In this work, I tried to consider these two facets separately which lead to the results on  $\Sigma$ -dags and asynchronous cellular automata on the one hand, and on recognizable and rational languages in divisibility monoids on the other hand. The general question whether these two lines can be merged again was posed by Wolfgang Thomas.

We list some more specific questions that are left open in the present work. For more details see the page indicated.

- Is the emptiness of  $L(\mathcal{A})$  for nonmonotone but effective asynchronous cellular machines decidable (page 41)? Furthermore, we did not consider the complexity of the emptiness problem for asynchronous cellular machines or automata.
- Is it decidable whether an asynchronous cellular machine accepts some Hasse-diagram (page 41)?
- For which sets of  $\Sigma$ -dags  $L$  is the set of  $\Sigma$ -ACAs  $\mathcal{A}$  with  $L(\mathcal{A}) = L$  recursive (page 54)?
- Is any complementable  $\Sigma$ -ACA equivalent to a deterministic ACA? (page 56)?
- Let  $k \in \mathbb{N}$ . Is the set of  $\Sigma$ -ACAs  $\mathcal{A}$  satisfying  $L(\mathcal{A}) \cap \mathbb{D}_k = L(\mathcal{A}_d) \cap \mathbb{D}_k$  for some deterministic  $\Sigma$ -ACA  $\mathcal{A}_d$  recursive (page 69)?
- Is there an extension of the monadic second order logic that allows one to axiomatize precisely the rational sp-languages (page 73)?
- Does there exist a divisibility monoid with infinitely many residuum functions (page 88)? If this is the case, is the property to have finitely many residuum functions decidable on input of a presentation as in Theorem 8.2.10?

- Is it possible to find finitely many sets  $C_q$  in a divisibility monoid such that rational sets where the iteration is applied to subsets of  $C_q$  only are recognizable (page 113)?
- We showed that any rational divisibility monoid with finitely many residuum functions is width-bounded. Is this implication valid without the assumption “finitely many residuum functions” (if there exists a divisibility monoid with infinitely many residuum functions at all, page 129)?
- Is the property to be width-bounded (i.e. to satisfy Kleene’s Theorem) decidable on input of a presentation as in Theorem 8.2.10?

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# Index

- $R(x)$ , 11
- $[a, b]$ , 3
- $[n]$ , 8
- $\mathcal{A}_M$ , 93
- $\mathcal{C}$ , 90
- $\mathbb{D}$ , 10
- $\Delta_A$ , 8
- $\mathbb{D}_k$ , 59
- $\mathbb{G}$ , 44
- $\mathbb{H}(A, \leq)$ , 3
- $\mathbb{H}_f(A, \leq)$ , 3
- $\mathbb{J}(A, \leq)$ , 3
- $\text{MATH}(V, \leq)$ , 7
- $\text{MATH}(\mathfrak{P})$ , 7
- $\text{MCTh}(V, \leq)$ , 7
- $\text{MCTh}(\mathfrak{P})$ , 7
- $\text{MTh}(V, E, \lambda)$ , 6
- $\text{MTh}(\mathbb{C})$ , 7
- $\text{Pred}(t)$ , 20
- $\text{SP}(\Sigma)$ , 71
- $S(\mathcal{A})$ , 25
- $\text{Th}(V, E, \lambda)$ , 6
- $\text{Th}(\mathbb{C})$ , 7
- $\bigvee(X)$ , 3
- $\bigwedge(X)$ , 3
- $\text{dom}(f)$ , 8
- $\downarrow X$ , 3
- $\text{fnf}(x)$ , 92
- $\text{id}_A$ , 8
- $\text{im } f$ , 8
- $\text{inf}(X)$ , 3
- $\text{mub}(X)$ , 3
- $\text{nat}$ , 84
- $\text{part}(A, B)$ , 8
- $\pi_i$ , 8
- $\text{state}(r)$ , 22
- $\text{sup}(X)$ , 3
- $\underline{n}$ , 8
- $\uparrow X$ , 1, 3
- $\uparrow a$ , 1
- $\uparrow\downarrow a$ , 3
- $a \prec c$ , 3
- $a \leq b$ , 3
- $a \parallel b$ , 3
- $h(a, A)$ , 3
- $k$ -chain mapping, 62
- $w(A, \leq)$ , 3
- $x \bowtie y$ , 85
- $x \uparrow y$ , 85
- $\Sigma$ -dag, 10
  - weak, 34
- $(\Sigma, k)$ -dag, 59
- a-regular, 77
- ACA, 13
- ACM, 12
  - effective, 27
  - monotone, 13
- alternating covering chain, 49
- antichain, 3
- asynchronous cellular automaton, 13
- asynchronous cellular machine, 12
- automaton over a monoid, 92
- axiomatizable
  - elementarily, 7
  - monadically, 7
- basis, 1
- basis algorithm, 27
- behavior, 93
- bound
  - largest lower, 3
  - least upper, 3
  - lower, 3
  - maximal lower, 3
  - minimal upper, 3
  - upper, 3
- branching automaton, 71
- cancellative, 82
- chain, 3
- chain covering, 59
- cliques, 90
- closed
  - residually, 113

- complementary, 85
- congruence, 4
- connected, 112
- convex, 3
- cover, 3
- definable, 7
- dependence alphabet, 4
- dependence graph, 5
- dependence relation, 4
- divisibility monoid, 84
- divisibility relation, 4
- filter, 3
  - principal, 3
- finitely generated, 81
- FNF, 92
- Foata normal form, 92
- formula
  - elementary, 6
  - monadic, 6
- free monoid, 4
- function
  - partial, 8
- generators, 81
- grid
  - complete, 106
  - folded, 44
- Hasse-diagram, 11
- height, 3
- Higman's Theorem, 2
- homomorphism, 4
- ideal, 3
  - principal, 3
- incomparable, 3
- independence relation, 4
- infimum, 3
- irreducible, 82
- join, 3
- join-irreducible, 3
- join-semilattice, 3
- Kleene's Theorem, 82
- language
  - weakly rational sp-, 72
  - closed word, 111
  - rational sp-, 72
  - series-rational sp-, 72
- width-bounded sp-, 72
- lattice, 3
  - distributive, 4
  - modular, 4
  - semimodular, 4
- left divisibility monoid, 84
- left divisibility relation, 4
- left divisor, 82
- length, 3
- lexicographical normal form, 118
- linearly ordered, 3
- Manfred Droste, 106
- meet, 3
- minimal, 1
- monoalphabetic, 83, 118
- monoalphabetic-rational language, 83
- monoid
  - labeled divisibility, 117
  - language
    - c-rational, 113
    - mc-rational, 118
    - rational, 124
- MSO, 6
- order, 3
- P-asynchronous automaton, 77
- Paul Gastin, 42
- Peter Habermehl, 19
- pomset
  - series-rational, 70
  - sp-, 70
- pomset without autoconcurrency, 11
- poset, 3
- product
  - parallel, 71
  - serial, 70
- quasi order, 1
- Ramsey's Theorem, 61
- rank, 111
- rational language, 82
- reading domain, 11
- recognizable language, 82
- relation
  - definable, 7
- relation defined by  $\varphi$ , 7
- residuum, 85
- residuum function, 86
- run, 14

- $\sim$  condition, 13
  - successful, 14
- sentence, 6
- spine, 61
- strongly equivalent, 100
- subgrid, 107
- supremum, 3
- theory
  - elementary, 6
  - monadic, 6
  - monadic antichain, 7
  - monadic chain, 7
- trace alphabet, 4
- trace monoid, 5
- transition
  - fork, 71
  - join, 71
  - sequential, 71
- transposed, 3
- tree width, 136
- well quasi order, 1
- well-structured transition system, 20
- width, 3
  - bounded, 123
- wqo, 1
- WSTS, 20